Name: ________________________________

Instructions

1. This exam is closed notes. Use of 3 standard 8.5 × 11 formulae sheets is permitted.
2. Time allotted is 2.5 hours.
3. Make sure that your reasoning is clearly stated. A considerable portion of the grades will be based on your reasoning. You should assume that I know nothing, and that everything must be explained to me.
4. Write neatly! If I can’t read it, I can’t grade it.
Problems

1. For the discrete-time plant: $P(z) = \frac{z + 1}{(z + 2)^2}$.

   Find a strictly-proper controller $C(z)$ such that all closed-loop poles are at the origin.
2. For the plant: \( P(s) = \frac{1}{(s - 1)^2} \).

(a) Write down the system in controllable-canonical form.
(b) Find a state-feedback controller that places all the closed-loop eigenvalues at \(-1\).
(c) Find an observer for this system with all observer poles at \(-2\).
3. Consider the plant: $P(s) = \frac{1}{s}$.

(a) Find the controller that minimizes

$$J(u) = \int_{0}^{\infty} y^2(t) + \rho u^2(t) \, dt, \quad \rho > 0.$$ 

(b) Determine how the poles of the closed-loop system behave as: $\rho \downarrow 0$ and $\rho \uparrow \infty$. Explain.
4. Consider a system with nominal model

\[ P_0(s) = \frac{1}{s} \]

and assume that the real system is given by

\[ P(s) = (1 + \Delta(s)) P_0(s) \]

(a) What condition must be satisfied by the complementary sensitivity function to ensure that the closed-loop system is stable?

(b) Suppose that

\[ |\Delta(j\omega)| < |W(j\omega)| \]

where \( W(s) = \frac{s + \sqrt{60}}{2s} \). Find a controller that ensures that the closed-loop system is stable for any such \( \Delta(s) \). **Hint.** Try a constant controller first.
5. Consider the continuous-time system consisting of the first order system:

\[ P(s) = \frac{1}{s + 1} \]

(a) Find the equivalent discrete-time plant assuming that the output is sampled every \( T \) seconds and the input is constant over intervals of length \( T \). Call this \( P_d(z) \).

(b) Find a discrete-time proper (not strictly-proper) controller that places the discrete-time poles at 0.