The Johns Hopkins University
Department of Electrical and Computer Engineering

505.460 — Introduction to Linear Systems — Fall 1997

Final exam

Name: _____________________________________________
Problems

1. What is the even component of the following signal?

\[ x(t) = 3u(t) + \sin(2\pi t) \]

2. Which of these properties does the system described by \( g(t) = x(t)x(t - 2)u(t) \) possess? Linear, invertible, memoryless, causal?
3. If $h[n] = u[n + 1] + \delta[n - 1]$ and $x[n] = \delta[n] + u[n - 1]$, then what is the output $y[n]$?

4. For a discrete-time LTI system with impulse response $h[n] = n\delta[n + 3]$ and input sketched below,

\[\begin{array}{c|cccc} h[n] \\ \hline -3 & -2 & -1 & 0 & 1 & 2 & 3 & n \end{array}\]

Compute the output
5. The Fourier series representation of a function has non-zero coefficients $a_0 = 2$, $a_3 = a_{-3} = -2$ and $a_1 = a_{-1} = j$ and period $T_0 = 1$. What is the function?

6. If $x(t) = 4 \cos t \sin t$, compute the Fourier transform.
7. Show how to construct an ideal band-pass filter with cut-off frequencies of 20 Hz and 100 Hz, if all you have is ideal low pass filters.

8. For the signal

\[ x(t) = 25 \sin(2\pi t) \cos(30\pi t) \]

What is the minimum sampling frequency that can be used to obtain samples of \( x(t) \) without loss of information?
9. In the double-side band modulation scheme:

\[ y(t) = x(t) \cos(2\pi f_m t) \]

and demodulation:

\[ z(t) = y(t) \cos(2\pi f_d t) \]

and \( w(t) = (h \ast z)(t) \).

What is the relationship between \( f_m, f_d \) and the bandwidth of \( x(t) \)? Also, what type of system is \( h \) if we are going to have \( w(t) = x(t) \)?

10. If the system \( y(t) = (h \ast x)(t) \) is described by

\[ \frac{dy(t)}{dt} = y(t - 1) + x(t) \]

What is the transfer function \( H(j\omega) \)?