Influence of Fibre Dispersion on the Performance of Optical Fibre Code Division Multiple Access Systems

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Abstract
Numerical simulation is frequently used to analyse direct sequence code division multiple access (DS/CDMA) systems and understand the influence on their performance of some important parameters. With this purpose, a Simulink/Matlab® based simulator has been developed for performance assessment of a DS/CDMA optical network. Considerable research effort has already been dedicated to this subject by others, but very few published results take into account some parameters which may be critical in high speed optical networks, namely the fibre dispersion and the non-linear of the laser sources. The study described here uses the moment generating function with the saddle point approximation for performance evaluation of DS/CDMA systems without neglecting those impairments.

I. INTRODUCTION
CDMA has been proposed for use in optical networks in 1985 [1], and a considerable research effort has been focused on this multiple access technique during the last decade. Several authors have used various analytical and numerical methods to evaluate the performance of this kind of systems, such as combinatorial analysis [2], Gaussian approximation [3], moment generating functions (MGF) [4], or numerical simulation [5, 6]. However, in spite of the generalised use of numerical simulation as a powerful tool for system performance assessment, a serious effort to take into consideration critical system parameters has not been done, in the studies dedicated to CDMA optical networks reported so far.

In a DS/CDMA system, each transmitted symbol is multiplied by a code sequence [4]. The performance measures of an IM/DD CDMA fibre link are directly related to the code sequence used by each transmitter. The selection of candidate sets of pulse sequences is basically a problem of code design, and has already been rigorously formulated, as in [7].

Due to its main characteristics, relatively low information rate and asynchronous transmission, optical CDMA is especially suitable for LANs [3]. Therefore, the main objective of the study reported here was to develop a numerical simulation tool with the possibility of handling some system parameters which are usually neglected in the analysis of CDMA systems, such as the effect of fibre dispersion and attenuation, the effect of laser non-linear response, and the effect of the receiver filter, for the estimation of system performance.

Actually, the tool which we have developed uses a semi-analytical approach. Firstly, a simulation method is used to describe the transmission of the optical signal through the network. After that, the MGF is used to describe the receiver statistics and to estimate the system performance. The MGF with saddle point approximation was chosen because it gives a very close approximation to the true probability of error [8].

II. SYSTEM MODEL

In an optical DS/CDMA communication system, there are L simultaneous sources (users) transmitting data asynchronously (Fig. 1). So, each of them produces an NRZ intensity modulated signal. A {0, 1}-valued periodic spreading waveform $C_j(n)$, with period $T = NT_c$ and chip duration $T_c$, is assigned to the $n$th user, representing the message symbol 1. For the symbol 0 no code is generated [4]. If the laser sources are assumed to be non coherent, then the intensity of the signal received by each user is the sum of the intensities contributed by the $L$ sources. Considering the usual star topology, the base band signal model generally considered in each branch of the network is:

$$s(t) = \sum_{n=1}^{L} \sum_{l=0}^{\infty} \sum_{j=-\infty}^{N-1} m_{ij}^{(n)} p_{ij} (t - lT_c - jT_c - \tau_n)$$

where $m_{ij}^{(n)} = c_j^{(n)} b_i^{(n)}$, $b_i^{(n)}$ is the $l$th equally probable data symbol of the $n$th user, $c_j^{(n)}$ is the $j$th chip of the $l$th
data symbol, $\tau_n$ is the $n$th user delay, and $p_{tr}$ is the pulse shape function.

The light pulses emitted by the $L$ transmitter lasers travel along the fibre and are collected by each optical receiver. At the end of the fibre, the light pulses are detected by an avalanche photodiode which generates primary electrons, which in turn produce secondary carriers by impact ionisation. If a light pulse corresponding to $m_{ij}^{(n)} = 1$ was transmitted, the signal received by user 1 would generate primary electrons with rate [4]:

$$\lambda = \frac{\Lambda_1 h(t)}{T_c}$$ (2)

with

$$\Lambda_1 = \frac{R P T_c}{q}$$ (3)

where $R$ is the photodiode responsivity (A/W), $P_c$ is the signal average power received in the chip interval $T_c$, $q$ is the electron charge, and $h(t)$ is the normalised fibre impulse response. Therefore, $\Lambda_1$ is the average number of primary electrons generated during an interval of duration $T_c$ if $h(t) = 1$.

When $c_{ij}^{(n)} = 0$, a smaller number of primary electrons, $\eta \Lambda_1$ (0 $\leq \eta < 1$), where $\eta$ is the extinction ratio, is generated because of the need to bias the laser above threshold, for high speed transmission. Another contribution to the primary current are the electrons due to dark current contribution to the primary current are the electrons due to

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Therefore, $\Lambda_1$ is the average number of primary electrons at the input of the APD is then:

$$\lambda(t) = \frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{j=0}^{\infty} \sum_{m=0}^{N-1} \lambda_{ij}^{(n)} h(t - jT_c - \tau_n) + \lambda_d$$ (4)

where:

$$\lambda_{ij}^{(n)} = (1 - \eta) m_{ij}^{(n)} + \eta$$ (5)

Equations (4) and (5) can be used to obtain:

$$\lambda(t) = (1 - \eta) \frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{j=0}^{\infty} \sum_{m=0}^{N-1} m_{ij}^{(n)} h(t - jT_c - \tau_n)$$

$$\lambda(t) = (1 - \eta) \frac{\Lambda_1}{T_c} \sum_{n=1}^{L} \sum_{j=0}^{\infty} \sum_{m=0}^{N-1} m_{ij}^{(n)} h(t - jT_c - \tau_n) + \lambda_d$$ (6)

Considering the statistical properties of the APD, Ho [4] obtained the following expression for the moment generating functions which describe the received signal $V(t)$ (for symbols “1” and “0”):

$$\mu_{b_0}^W(s) = \exp \left\{ \frac{1}{2} \sigma^2 s^2 + \frac{1}{2} \sum_{j=1}^{L} F_{bj} D_j(s) + w b_{ij}^{(n)} D_{ij}(s) \right\} \prod_{j=1}^{L} \Gamma_j(s)$$ (7)

where $\sigma^2$ is the thermal noise variance.

To obtain the probability of error of the system we have to compute the following equation:

$$P_e = \frac{1}{2} f_0(V_{th}) + \frac{1}{2} f_1(V_{th})$$ (8)

with:

$$f_0(V_{th}) = \Pr[V(t) \geq V_{th}, b_0^{(n)} = 0]$$

$$f_1(V_{th}) = \Pr[V(t) < V_{th}, b_0^{(n)} = 1]$$ (9)

where $f_i$ are the probability density functions associated with the detection process $V(t)$ when the transmitter sends a one ($i = 1$) or a zero ($i = 0$), which may be obtained by:

$$f_k(V_{th}) = \int_{-\infty}^{\infty} e^{\psi_k(s)} ds / 2\pi i, \quad k = 0,1$$ (10)

where

$$\psi_k(s) = \ln \mu_k(s) - s V_{th} - \ln \left[ \left( -1 \right)^k s \right]$$ (11)

Since the solution of equation (10) is non-trivial, an approximation is usually used. Ho [4] considered the Gaussian and the saddle point approximations, and he has shown that the Gaussian approximation cannot provide a good approximation to the probability of error in CDMA systems, but that the saddle point approximation produces a result very close to the true value.

The saddle point approximation for the probability density functions is obtained with the following equation:

$$f_k(V_{th}) = \frac{1}{\sqrt{2\pi \psi_k'(s_k)}} \exp[\psi_k(s_k)]$$ (12)

The saddle point is the solution of the equation:

$$\psi_k(s) = 0$$ (13)

with:

$$\psi_k(s) = \ln \mu_k(s) - s V_{th} - \ln \left[ \left( -1 \right)^k s \right]$$ (14)

The Newton method is used to solve equation (14). The calculation is iterated until the variation of the calculated value falls below $10^{-5}$. An initial trial value is selected and at each stage of iteration we obtain a new trial value given by:

$$M \leftarrow M - \frac{\psi_k(s)}{\psi_k'(s)} M'(s)$$ (15)

where the second derivative is given by:

$$\psi_k''(s) = \left[ \ln \mu_k(s) \right]' + s^{-2}$$ (16)

As it is well known, in IM/DD systems the threshold $V_{th}$ may have influence on the probability of error. So, it is necessary to determine an optimum decision level, which may be obtained by solving:

$$f_0(V_{th}) = f_1(V_{th})$$ (17)

or, equivalently, the equation:

$$R(V_{th}; \Lambda_1) = \ln f_0(V_{th}) - \ln f_1(V_{th})$$ (18)

Ho [4] suggested the use of the secant method to solve this equation. At each stage, a trial value of $V_{th}$ is obtained by the iteration:
where \( V_{Ts} \) is the value used is:
\[
V_{Ts} \approx \frac{m_0 + m_1}{2} = G\Lambda_w \left( m_d + m_i + \frac{1}{2} \right)
\]
where \( m_0, m_1, m_d, \) and \( m_i \) are defined in [4].

The simulator has two purposes: one is to simulate the generation, transmission, and reception of the signal emitted by a particular source (user), the other is to compute the system performance in the presence of interfering signals and system impairments such as those above referred.

III. NUMERICAL RESULTS

To assess the system performance taking into consideration the influence of the source response and optical fibre propagation characteristics, we use the sequence of light pulses generated by driving the laser with a current wave form obtained by the multiplication of the data symbol by the selected code. This is propagated along the optical fibre, and processed by the optical receiver. The simulator has two purposes: one is to simulate the generation, transmission, and reception of the signal emitted by a particular source (user), the other is to compute the system performance in the presence of interfering signals and system impairments such as those above referred.

The simulator computes the received signal taking into account the effect of exponential rising and falling edges of the NRZ current driver, the real shape of the pulses emitted by the laser source, the fibre dispersion, and the transfer function of the optical receiver (modelled by a third-order Butterworth filter). The probability of error is estimated by solving equation (8).

A DFB laser with a chip rate of 10 Gchip/s and three codes, two prime codes with lengths 529 and 121 and an orthogonal optical code (OOC) with length 127, have been considered to obtain the results presented here. With these parameters, the system has an overall channel data rate of 18.9 Mbit/s, 82.6 Mbit/s, and 78.7 Mbit/s for each code, respectively. In order to have a high extinction ratio, the
laser was considered as being polarised close to threshold. To determine the extinction ratio used in the estimation of error probability, we have used equation (22). Since the system is subjected to power loss mechanisms, namely coding and fibre attenuation, we have considered the possibility of several values for the received power. The receiver parameters used for simulation can be found in [4].

The results shown in Fig. 3 were obtained considering the system operating at chip rate of 10 Gchip/s, using a prime code of length 121, for three values of fibre length, $L_f$, and with 2 and 5 interfering users.

We considered also the case of using a prime code with length 529. As we can observe in Fig. 4, for the same received power we have a lower probability of error. The performance improvement is due to the increase of code length. Besides, the penalty due to the fibre propagation characteristics is less significant for this longer code. The reason for this is that, in the longer code, the relation between the code length and the chip number (code height) is larger. Therefore, the inter-chip interference (ICI) is less significant.

Another aspect that we may observe in these results is that the influence of fibre length depends on the number of interfering users.

We have also made a comparison of the performance obtained using an optical orthogonal code (OOC) [7], and a prime code with approximately the same length. This assumption guarantee that the chips are subjected to the same ICI. In the analysis we assumed an OOC with length 127 and a prime code with length 121. When we consider two interfering users in the system, we obtain the results shown in Fig. 5.

We can conclude that the prime code leads to a better performance and is more sensitive to the length of the optical fibre. The performance improvement in this case is due to the fact that we have considered the same power for each chip and the prime code has 11 chips, whereas the OOC has only 3 chip. Besides, due to the larger relation between length and chip number in the OOC, this code is less sensitive to the fibre length than the prime code.
IV. CONCLUSION

We have applied a new methodology for performance evaluation of optical DS/CDMA systems: the combination of signal transmission simulation with the MGF method for estimation of the error probability (BER).

In earlier studies, as far as we know, only one of these strategies has been used: both the computation and characterisation of the received signal [5, 6], without estimation of BER, or the performance estimation assuming a given signal at the input of the receiver [1-4]. The latter used either the MGF or the Gaussian approximation methods.

A simulator has been developed which enables the study of many critical parameters of an optical CDMA system.

From our results, we can conclude that the available technology allows the implementation of an optical network, with acceptable values for the probability of error, where the number of simultaneous users may be limited by the performance of some critical subsystem parameters.

Usually, in the analysis of the codes used in DS/CDMA, the main objective is to find the code set with the best correlation properties. From our results, we may conclude that the codes used in this kind of systems must be selected taking into consideration not only their correlation properties, but also their dispersion tolerance. Besides, we have found that the dispersion tolerance depends on the relation between the code length and number of chips in the particular coding set used by the system. The influence of this effect increases with the chip rate.

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VI. REFERENCES


