520.485 Chapter 2

MOBILITY

$J_{drift} = J_{n,d} + J_{p,d} = -env_{d,n} + epv_{d,p}$

In general,

$v_{n,d} = v_{n,d}(\mathrm{E}) = -\mu_n \mathrm{E};\ v_{n,d} = v_{p,d}(\mathrm{E}) = \mu_p \mathrm{E}$

thus

$J_{drift} = -en\mu_n \mathrm{E} + ep\mu_p \mathrm{E}$

To find mobility:

$$\frac{dv_n}{dt} = -\frac{e}{m_c}\mathrm{E};\ v_n(t) = -\frac{et_c}{m_c}\mathrm{E};$$

where t_c time from the last collision. Perform averaging

$$\langle v_n \rangle = -\left\langle \frac{et_c}{m_c}\mathrm{E} \right\rangle = -\frac{e\langle t_c \rangle}{m_c}\mathrm{E} = -\frac{e\tau_{c,n}}{m_c}\mathrm{E};$$

In case of the multi-valley conduction band we need to perform averaging over six valleys. Then $\mu_n = \frac{e}{m_n}\tau_{c,n}; \sim \frac{1}{m_n} = \frac{1}{3}\left(\frac{1}{m_L} + \frac{2}{m_T}\right)$ where we introduced conductivity effective mass $m_n \sim 0.26 m_0$ (Si)

For valence band we need to weigh over the number of heavy holes and light holes, that relate as their density of states, thus $\mu_p = \frac{e}{m_p}\tau_{c,p}; \sim \frac{1}{m_p} = \frac{m_{hh}^{1/2} + m_{lh}^{1/2}}{m_{hh}^{3/2} + m_{lh}^{3/2}}$, For Si $m_p \sim 0.4 m_0$

Collision times:

Phonon scattering

$$\mu_L \sim \tau_{c,L} \sim T^{-3/2}$$

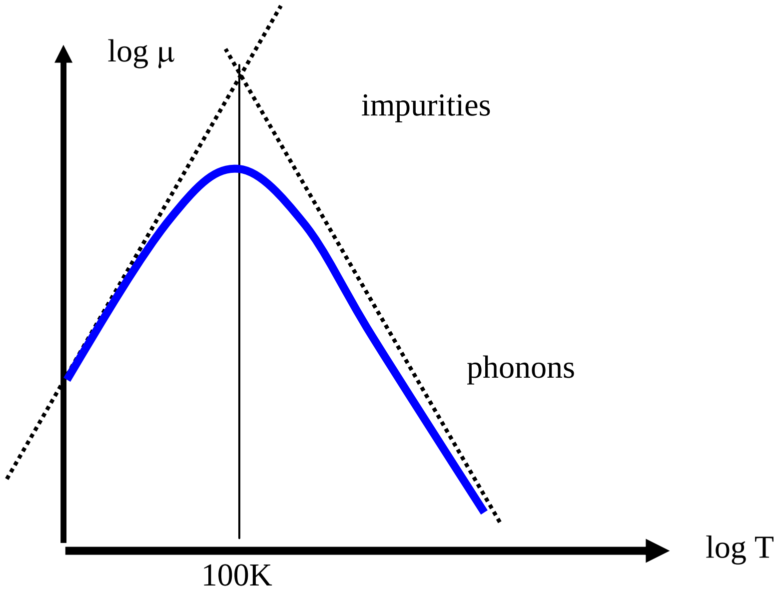
Ionized impurity scattering

$$\mu_I \sim \tau_{c,I} \sim T^{3/2} N_I^{-1}$$

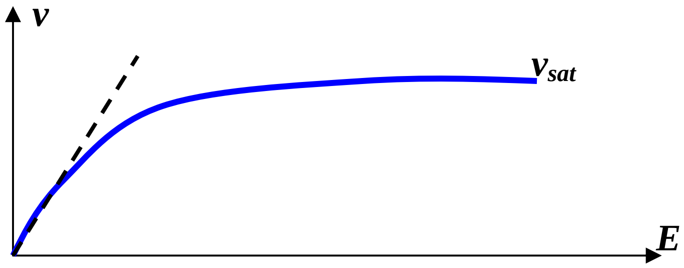
total mobility $\mu^{-1} = \mu_L^{-1} + \mu_I^{-1}$

$\mu_{n,Si} \sim 10^4 cm^2 V^{-1} s^{-1}$

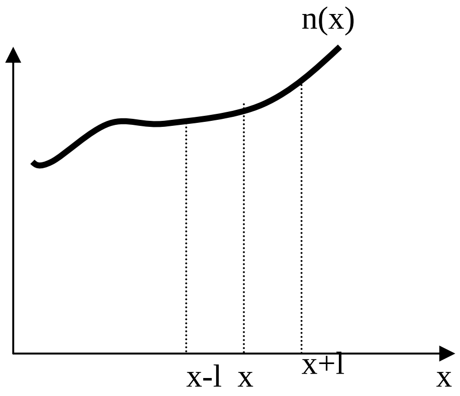
$\mu_{n,GaAs} \sim 10^5 cm^2 V^{-1} s^{-1}$



At high field mobility gets lower – eventually – velocity saturation.



DIFFUSION



Mean free path: $l_n = v_{n,th}\tau_{c,n}$

Current density due to the thermal motion from left to right:

$$J_{n,1} = -\frac{e}{2} n(x - l_n) v_{n,th}$$

Current density due to the thermal motion from right to left:

$$J_{n,2} = +\frac{e}{2} n(x + l_n) v_{n,th}$$

Net current density:

$$J_{n,diff} = J_{n,1} + J_{n,2} = -\frac{e}{2}n(x-\mathrm{l}_{n,x})v_{n,th,x} + \frac{e}{2}n(x+\mathrm{l}_{n})v_{n,th,x} =$$

$$ev_{n,th}l_{n,x}\frac{n(x+l_{n,x})-n(x+l_{n,x})}{2l_n} \approx ev_{n,th,x}l_{n,x}\frac{dn}{dx} = ev^2_{n,th,x}\tau_{c,n}\frac{dn}{dx}$$

Average over the directions: $\left\langle v^2_{n,th,x}\right\rangle = \frac{1}{3}v^2_{n,th} = \frac{kT}{m_n}$

$$J_{n,diff} = eD_n\frac{dn}{dx};\ D_n = \frac{kT}{m_n}\tau_{c,n} = \frac{kT}{e}\mu_n$$

Total current

$$J_n = J_{n,drf} + J_{n,diff} = e\mu_n n\mathrm{E} + eD_n\frac{dn}{dx}$$

$$J_p = J_{p,drf} + J_{p,diff} = e\mu_p p\mathrm{E}\text{-}eD_n\frac{dp}{dx}$$

GENERATION AND RECOMBINATION OF CARRIERS

Radiative and nonradiative recombination without traps –bimolecular recombination. Can be radiative and non-radiative.

Two processes

Generation - $G(T)$
Recombination - $R = Bnp$
At equilibrium $G = R = Bn_0p_0 = Bn_i^2$
In non-equilibrium

$$\frac{dn}{dt} = \frac{dp}{dt} = G - B(n_0+\delta n)(p_0+\delta p) = -B(n_0\delta p + p_0\delta n + \delta p\delta n)$$

Intrinsic or depleted semiconductor strong injection $n_0 \sim p_0 \le n_i << \delta n = \delta p$

$$\frac{d\delta n}{dt} = \frac{d\delta p}{dt} = -B\delta p\delta n = -B\delta n^2$$ bi-molecular recombination

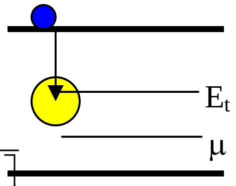
Highly doped semiconductor, low injection $\delta n = \delta p << n_0 \sim N_d$

$$\frac{d\delta n}{dt} = \frac{d\delta p}{dt} = -Bn\delta p = -\frac{\delta p}{\tau_p}$$

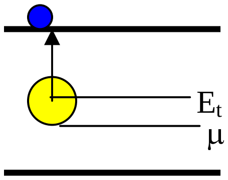
SRH –RECOMBINATION ON TRAPS

There are four processes

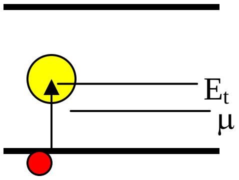
1. Electron’s capture $R_{c,n} = v_n \sigma_n N_t (1 - f_F(E_t))n;\ f_F(E_t) = \dfrac{1}{1 + \exp\left[\dfrac{E_t - \mu}{kT}\right]}$



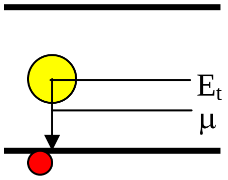
2. Electron’s excitation $R_{e,n} = X_n N_t f_F(E_t)$



3. Hole’s capture $R_{c,p} = v_p \sigma_p N_t f_F(E_t)p;\ f_F(E_t) = \dfrac{1}{1 + \exp\left[\dfrac{E_t - \mu}{kT}\right]}$



4. Hole’s excitation $R_{e,p} = X_p N_t (1 - f_F(E_t))$



At equilibrium

$$R_{c,n} = v_n \sigma_n N_t (1 - f_{F0}(E_t)) n_0 = R_{e,n} = X_n N_t f_{F0}(E_t)$$

$$X_n = v_n \sigma_n \left[f_{F0}^{-1}(E_t) - 1 \right] N_c \exp\left[-\frac{E_g - \mu}{kT} \right] = v_n \sigma_n \exp\left[\frac{E_t - \mu}{kT} \right] N_c \exp\left[-\frac{E_g - \mu}{kT} \right] =$$

$$= v_n \sigma_n \exp\left[-\frac{E_g - E_t}{kT} \right] N_c = n' v_n \sigma_n. \qquad n' = N_c \exp\left[-\frac{E_g - E_t}{kT} \right]$$

and for holes

$$R_{c,p} = v_p \sigma_p N_t f_{F0}(E_t) p_0 = R_{e,p} = X_p N_t (1 - f_{F0}(E_t))$$

$$X_p = v_p \sigma_p \left[f_{F0}^{-1}(E_t) - 1 \right]^{-1} N_v \exp\left[-\frac{\mu}{kT} \right] = v_p \sigma_p \exp\left[-\frac{E_t - \mu}{kT} \right] N_v \exp\left[-\frac{\mu}{kT} \right] =$$

$$= v_p \sigma p \exp\left[-\frac{E_t}{kT} \right] N_c = p' v_p \sigma_p. \qquad p' = N_v \exp\left[-\frac{E_t}{kT} \right]; \ \textbf{Note}: n' p' = n_i^2$$

In non-equilibrium: net electron recombination

$$R_n = R_{c,n} - R_{e,n} = v_n \sigma_n N_t [1 - f_F(E_t)] n - X_n N_t f_F(E_t) =$$
$$v_n \sigma_n N_t [1 - f_F(E_t)] n - n' v_n \sigma_n N_t f_F(E_t) =$$
$$v_n \sigma_n N_t \left[[1 - f_F(E_t)] n - n' f_F(E_t) \right]$$

and net hole recombination

$$R_p = R_{c,p} - R_{e,p} = v_p \sigma_p N_t f_F(E_t) p - X_n N_t (1 - f_F(E_t)) =$$
$$v_p \sigma_p N_t \left[p f_F(E_t) - p'(1 - f_F(E_t)) \right]$$

Recombination rates for electrons and holes are equal

$$R_n = v_n \sigma_n N_t \left[(1 - f_F(E_t)) n - n' f_F(E_t) \right] = v_p \sigma_p N_t \left[p f_F(E_t) - p'(1 - f_F(E_t)) \right] = R_p$$

$$f_F(E_t) = \frac{v_n \sigma_n n + p' v_p \sigma_p}{v_n \sigma_n (n + n') + v_p \sigma_p (p + p')}$$

$$R_n = R_p = \frac{v_n \sigma_n v_p \sigma_p (np - n_i^2)}{v_n \sigma_n (n + n') + v_p \sigma_p (p + p')}$$

LOW INJECTION

$$n = n_0 + \delta n;\ p = p_0 + \delta p;\ n_0 p_0 = n_i^2$$

$$R_n = R_p = \frac{v_n \sigma_n v_p \sigma_p N_t (n_0 \delta p + p_0 \delta n)}{v_n \sigma_n (n_0 + n') + v_p \sigma_p (p_0 + p')}$$

First consider intrinsic material with simultaneous injection of electrons and holes $\delta n = \delta p;\ n_0 = p_0 = n_i$. We obtain

$$R = \frac{2 v_n \sigma_n v_p \sigma_p N_t n_i \delta n}{v_n \sigma_n (n_i + n') + v_p \sigma_p (n_i + n_i^2 / n')} = \frac{\delta n}{\dfrac{1 + n'/n_i}{2 v_p \sigma_p N_t} + \dfrac{1 + n_i / n'}{2 v_n \sigma_n N_t}} = \frac{\delta n}{\tau}$$

Let us minimize τ

Easy to see that

$$n'_{opt} = n_i \sqrt{\frac{v_p \sigma_p}{v_n \sigma_n}};\ \ N_c \exp\left[-\frac{E_g - E_t}{kT}\right] = N_c \exp\left[-\frac{E_g - E_i}{kT}\right] \sqrt{\frac{v_n \sigma_n}{v_p \sigma_p}}$$

$$E_{t,opt} = E_i + \frac{kT}{2} \ln \frac{v_p \sigma_p}{v_n \sigma_n};\ \ \tau_{\min} = \frac{1}{2}\left[\frac{1}{\sqrt{v_n \sigma_n N_t}} + \frac{1}{\sqrt{v_p \sigma_p N_t}}\right]^2 = \frac{1}{2}\left[\frac{1}{\sqrt{\tau_p}} + \frac{1}{\sqrt{\tau_n}}\right]^2$$

For the intrinsic semiconductor the recombination on traps is most effective if the traps are located close to the mid-gap!!!

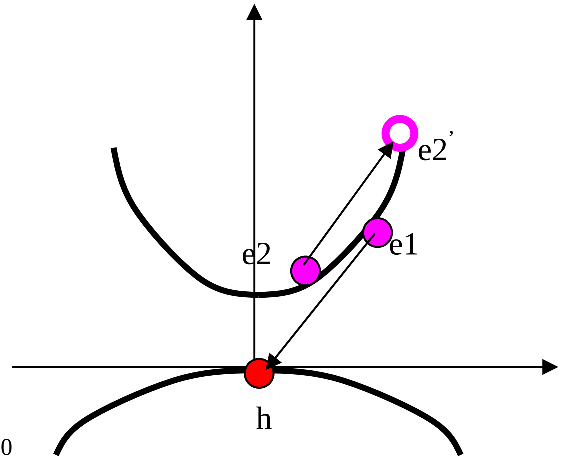
Next consider strongly doped n-type semiconductor

$$R = v_p \sigma_p N_t \delta p = \frac{\delta p}{\tau_p}.$$

or strongly-doped p-type semiconductor

$$R = v_n \sigma_n N_t \delta n = \frac{\delta n}{\tau_n}$$

In the doped material the recombination rate is determined by the minority carriers.



AUGER RECOMBINATION

(process inverse to the impact ionization)

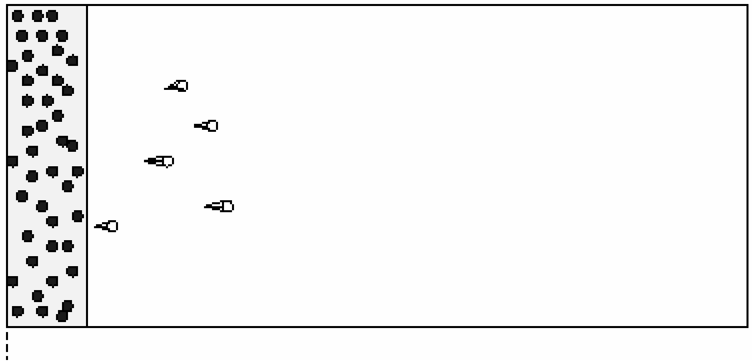
Very strong injection $\delta n \sim \delta p >> n_0, p_0$

Tri-molecular process $\frac{dn}{dt} \sim C_1 np^2 + C_2 pn^2 \sim C\delta n^3$

In general, for the excess carriers we can introduce effective recombination time

$$\frac{d\delta n}{dt} = A\delta n + B\delta n^2 + C\delta n^3 = \frac{\delta n}{\tau_n}$$

SURFACE RECOMBINATION



Traps are all located in the thin surface layer with two-dimensional (sheet) density $N_t^{(2D)}$. Then one can introduce surface recombination velocity for the minority carriers, say electrons: $S_n = \sigma_n v_n N_t^{(2D)}$. The boundary condition is therefore:

$$D_n \frac{\partial n}{\partial x}\Big|_{x=0} = S_n\left[n(0) - n_0\right]$$

CONTINUITY EQUATIONS

$$\frac{\partial n}{\partial t}=\frac{1}{e}\frac{\partial J_n}{\partial x}-\frac{n-n_0}{\tau_n}=\frac{1}{e}\frac{\partial J_{n,dif}}{\partial x}+\frac{1}{e}\frac{\partial J_{n,drift}}{\partial x}-\frac{n-n_0}{\tau_n}=D_n\frac{d^2n}{dx^2}+\mu_n\mathrm{E}\frac{dn}{dx}+\mu_n n\frac{d\mathrm{E}}{dx}-\frac{n-n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t}=-\frac{1}{e}\frac{\partial J_p}{\partial x}-\frac{p-p_0}{\tau_n}=-\frac{1}{e}\frac{\partial J_{p,dif}}{\partial x}-\frac{1}{e}\frac{\partial J_{p,drift}}{\partial x}-\frac{p-p_0}{\tau_n}=D_p\frac{d^2p}{dx^2}-\mu_p\mathrm{E}\frac{dn}{dx}+\mu_p p\frac{d\mathrm{E}}{dx}-\frac{p-p_0}{\tau_n}$$

AMBIPOLAR DIFFUSION

Continuity equations are:

$$\frac{\partial n}{\partial t}=D_n\frac{\partial^2 n}{\partial x^2}+\mu_n(\mathrm{E}\frac{\partial n}{\partial x}+n\frac{\partial\mathrm{E}}{\partial x})+g_n-R_n;$$

$$\frac{\partial p}{\partial t}=D_p\frac{\partial^2 p}{\partial x^2}-\mu_p(\mathrm{E}\frac{\partial p}{\partial x}+p\frac{\partial\mathrm{E}}{\partial x})+g_p-R_p;$$

The generation-recombination terms are equal for the electrons and holes. Therefore, if we subtract two equations we obtain

$$\frac{\partial(\delta p-\delta n)}{\partial t}=D_p\frac{\partial^2\delta p}{\partial x^2}-D_n\frac{\partial^2\delta n}{\partial x^2}-\mu_p(\mathrm{E}\frac{\partial\delta p}{\partial x}+p\frac{\partial\mathrm{E}}{\partial x})-\mu_n(\mathrm{E}\frac{\partial\delta n}{\partial x}+n\frac{\partial\mathrm{E}}{\partial x})$$

Now, there is an internal field of mutual attraction and repulsion of the excess carriers

$$\frac{\partial\mathrm{E}_{\mathrm{int}}}{\partial x}=\frac{e}{\varepsilon}\left(\delta p-\delta n\right).$$

Substitute and use the Einstein relations

$$\frac{\partial(\delta p-\delta n)}{\partial t}=\left(\mu_p\frac{kT}{e}\frac{\partial^2\delta p}{\partial x^2}-\mu_n\frac{kT}{e}\frac{\partial^2\delta n}{\partial x^2}\right)-\left(\mu_p\mathrm{E}\frac{\partial\delta p}{\partial x}+\mu_n\mathrm{E}\frac{\partial\delta n}{\partial x}\right)-\frac{e}{\varepsilon}\left(\mu_p p+\mu_n n\right)(\delta p-\delta n)$$

Let us now compare the orders of magnitude. Assume $n>>p$. If the characteristic dimension over which the change in the carrier concentration occurs is $\delta x\geq 1\mu m$ and the voltage drop over that distance is $\delta V\leq 0.1V$ then we can see that the first term has an order of magnitude of

$\frac{\mu_n kT}{e(\delta x)^2} \le 2.5\times 10^6 \mu_n / s$, the second term has an order of magnitude $\frac{\mu_n \delta V}{(\delta x)^2} \le 10^7 \mu_n / s$, while the third term has order magnitude $2\times 10^{-7} n\mu_n / s$. Therefore, for $n > 10^{15} cm^{-3}$ the third term dominates and we can write

$$\frac{\partial(\delta p - \delta n)}{\partial t} = -\frac{(\delta p - \delta n)}{\tau_d}$$

where $\tau_d = \frac{\varepsilon}{e(n\mu_n + p\mu_p)} = \frac{\varepsilon}{\sigma}$ **-dielectric relaxation time**

Typically this time is less than 1ps. Therefore, we can use quasi-neutrality condition $\delta n \approx \delta p$. Then

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} + \mu_n (\mathrm{E}\frac{\partial \delta n}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}) + g - R; \ \times \mu_p p$$

$$\frac{\partial \delta n}{\partial t} = D_p \frac{\partial^2 \delta n}{\partial x^2} - \mu_p (\mathrm{E}\frac{\partial \delta n}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}) + g - R; \ \times \mu_n n$$

add

$$(\mu_n n + \mu_p p)\frac{\partial \delta n}{\partial t} = (\mu_n n D_p + \mu_p p D_n)\frac{\partial^2 \delta n}{\partial x^2} + \mu_n \mu_p (p - n)\mathrm{E}\frac{\partial \delta n}{\partial x} + (\mu_n n + \mu_p p)(g - R)$$

divide by $(\mu_n n + \mu_p p)$

$$\frac{\partial \delta n}{\partial t} = D' \frac{\partial^2 \delta n}{\partial x^2} + \mu' \mathrm{E}\frac{\partial \delta n}{\partial x} + g - R$$

where $D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$ and $\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$

and we have used Einstein relations

Now consider the n-type semiconductor and low injection $n >> p$ - $D' = D_p; \quad \mu' = \mu_p$ **it is minority carrier that determines the diffusion, just as it is minority carrier that determines recombination rate.**

SCALING

Now it is important to understand the scales, in time and space, on which the diffusion takes place. Let us consider one-dimensional problem of continuity equation

$$\frac{\partial n}{\partial t} = \frac{n_0 - n}{\tau_n} + D_n \frac{\partial^2 n}{\partial x^2};\ n(0) = 0; n(\infty) = n_0 \vdots$$

First of all, let us change the scale to
$\tau = t/\tau_n;\ \xi = x/L_n$ where $L_n = \sqrt{D_n \tau_n} - \textit{diffusion length}$
Now the continuity equation becomes

$$\frac{\partial(n\text{-}n_0)}{\partial \tau} = \text{-}(n - n_0) + D_n \frac{\partial^2 (n\text{-}n_0)}{\partial \xi^2} = 0$$

The steady-state solution is $(n - n_0) = -n_0 e^{-\xi} = -n_0 e^{-x/L_n}$
Meaning – the disturbance at x=0 spreads only about one diffusion length. What is the diffusion current at *x=0*?
$J_n = eD_n \frac{dn}{dx} = e\frac{D_n}{L_n} n_0 e^{-x/L_n}$ at *x=0* we have $J_n = e\frac{D_n}{L_n} n_0 = e\frac{L_n}{\tau_n} n_0$
It all makes sense – only the electrons that are within one diffusion length from the boundary “see it” and it takes them one recombination time to get there. In general, one can talk about “diffusion velocity” $v_{n,diff} = L_n/\tau_n = \sqrt{\frac{D_n}{\tau_n}}$

But what happens when we have a different situation: the boundary conditions are

$$n(0) = 0;\ n(d) = n_0;\ d << L_n\ .$$

We can almost guess our solution $n = \frac{x}{L} n_0$

Then we obtain $J_n = eD_n \frac{dn}{dx} = e\frac{D_n}{L} n_0$ and now $v_{n,diff} = D_n/L$ and the time that it takes for the electron to traverse the thickness L is $\tau_{diff} = L/v_{n,diff} = L^2/D_n$ - this is the time scale on which diffusion takes place in a small sample.