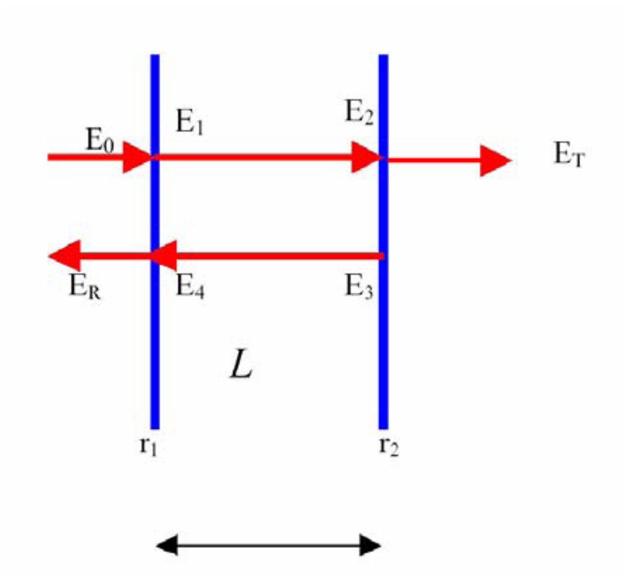
Introduction to Lasers (520.482)

Solutions: Homework 3

#1. (40)



The solution to this problem in the presence of absorption (α) or gain (γ) can be found out in a way similar to as derived in the Lecture 8 notes. The only difference would be that there would be an additional absorption/gain term given by:

$$\theta' = k'L = \left(k_0 + j\frac{\alpha}{2}\right)L$$

So if we make the above substitution and solve for it (for the case of absorption), we get:

$$E_R = \frac{r_1' + r_2 e^{j2\theta'}}{1 - r_1 r_2 e^{j2\theta'}} E_0 = \frac{r_1' + r_2 e^{j2\theta} e^{-\alpha L}}{1 - r_1 r_2 e^{j2\theta} e^{-\alpha L}} E_0$$

Now using $r_1' = -r_1$, $r_1^2 + t_1^2 = 1$, reflection coefficient of the Fabry Perot resonator can be found out by calculating the ratio of the reflected field intensity and incident field intensity.

$$R = \left|\frac{E_R}{E_0}\right|^2 = \left|\frac{r_1' + r_2 e^{j2\theta} e^{-\alpha L}}{1 - r_1 r_2 e^{j2\theta} e^{-\alpha L}}\right| = \frac{R_1 + R_2 e^{-2\alpha L} - 2\sqrt{R_1 R_2} e^{-\alpha L}\cos(2\theta)}{1 + R_1 R_2 e^{-2\alpha L} - 2\sqrt{R_1 R_2} e^{-\alpha L}\cos(2\theta)}$$

Transmission can be found by simply using the fact that $R + T = 1$, and hence,

$$T = \frac{1 + R_1 R_2 e^{-2\alpha} - R_1 - R_2 e^{-2\alpha L}}{1 + R_1 R_2 e^{-2\alpha} - 2\sqrt{R_1 R_2} e^{-\alpha L}\cos(2\theta)} = \frac{(1 - R_1)(1 - R_2 e^{-2\alpha L})}{1 + R_1 R_2 e^{-2\alpha L} - 2\sqrt{R_1 R_2} e^{-\alpha L}\cos(2\theta)}$$

or $I_T = \dfrac{(1-R_1)(1-R_2e^{-2\alpha L})}{\left(1-\sqrt{R_1R_2}e^{-\alpha L}\right)^2 + 4\sqrt{R_1R_2}e^{-\alpha L}\sin^2\theta} I_0$

Now to find the FWHM of the transmission curve, we need to find out the points where

$$(I_T) = \frac{(I_T)_{\max}}{2}$$

$$\sin\theta_{\pm} = \frac{1-e^{-\alpha L}\sqrt{R_1R_2}}{2e^{-\frac{\alpha L}{2}}\sqrt[4]{R_1R_2}} = \pi\Delta\nu_{1/2} n L/c$$

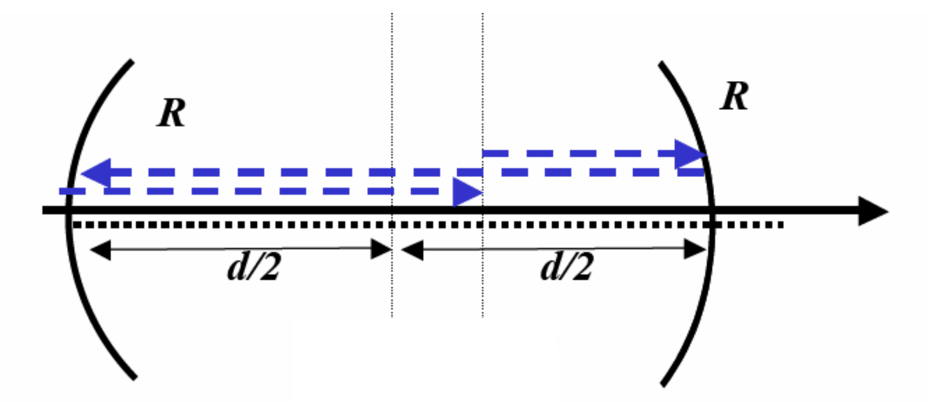
the FWHM of transmission curve is then given by: $\Delta\nu_{1/2} = \dfrac{c}{2\pi nL}\dfrac{1-e^{-\alpha L}\sqrt{R_1R_2}}{e^{-\frac{\alpha L}{2}}\sqrt[4]{R_1R_2}}$

Finesse is defined by $F = \dfrac{FSR}{\Delta\nu_{1/2}}$.

Hence Finesse for a Fabry Perot etalon with absorption is given by: $F = \dfrac{\pi e^{-\frac{\alpha L}{2}}\sqrt[4]{R_1R_2}}{1-e^{-\alpha L}\sqrt{R_1R_2}}$.

To derive for Gain medium just substitute $\alpha = -\gamma$

#2. (10)



Laser cavity could be modeled as a cavity with two symmetric spherical mirrors (Lecture 10, p.9) where, $-1 \le 1 - d/R \le 1$ or $0 \le d/R \le 2$.

At the beam waist,

$$\omega^2(0) = \frac{\lambda d}{2\pi n}\sqrt{2R/d - 1}.$$

At the mirror,

$$\omega^2(d/2) = \frac{\lambda d}{\pi n} \frac{1}{\sqrt{(d/R)(2-d/R)}} .$$

(Assume n = 1.)