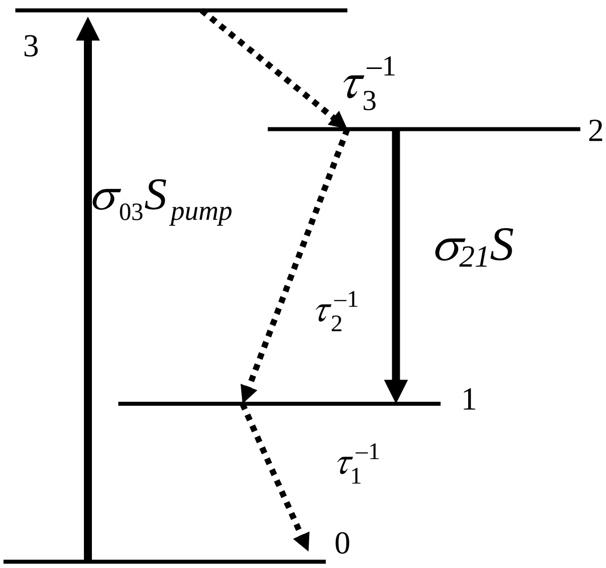
Lecture 7
Laser Amplifiers

Gain Saturation



Consider now the four level laser system with very low values of times τ_1 and τ_3 in the presence of both pump S_{pump} and signal S. Rate equations are

$$\frac{dN_2}{dt} = \sigma_{13}\frac{S_{pump}}{\hbar\omega_{pump}}\left(N - N_2\right) - \frac{N_2}{\tau_2} - \sigma_{21}\frac{S}{\hbar\omega}N_2$$

In the absence of signal

$$N_2^0 = N\frac{\sigma_{03}S_{pump}/\hbar\omega_{pump}}{\sigma_{03}S_{pump}/\hbar\omega_{pump} + \tau_2^{-1}} = \frac{\sigma_{03}S_{pump}}{\hbar\omega_{pump}}\tau_2'$$

where the effective lifetime (or gain recovery time) gets is modified by pump saturation

$$\frac{1}{\tau_2'} = \frac{1}{\tau_2} + \frac{\sigma_{03}S_{pump}}{\hbar\omega_{pump}}$$

So, with the signal present we obtain

$$N_2(S) = N\frac{\sigma_{03}S_{pump}/\hbar\omega_{pump}}{\sigma_{03}S_{pump}/\hbar\omega_{pump} + \tau_2^{-1} + \sigma_{21}S/\hbar\omega} = \frac{N_2^{(0)}}{1 + S/S_{sat}}$$

where the saturation power density is

$$S_{sat} = \hbar\omega / \sigma_{21}\tau_2'$$

Now the gain itself is saturated and we can write

$$\gamma(S) = \frac{\sigma_{21}N_2^0}{1+S/S_{sat}} = \frac{\gamma_0}{1+S/S_{sat}}$$

Power broadening

Now, let us take into account the frequency dependence of the stimulated emission cross-section

$$\sigma_{21}(\omega) = \frac{\sigma_{21}(\omega_0)}{1+\left(2\frac{\omega_0-\omega}{\Delta\omega_h}\right)^2}$$

Therefore

$$S_{sat}(\omega) = S_{sat}(\omega_0)\left[1+\left(2\frac{\omega_0-\omega}{\Delta\omega_h}\right)^2\right]$$

and for the saturated gain shape that we obtain by scanning the beam with power S over the spectrum we obtain

$$\gamma(S,\omega) = \frac{\gamma_0(\omega)}{1+S/S_{sat}(\omega)} = \frac{\gamma_0(\omega_0)}{1+\left(2\frac{\omega_0-\omega}{\Delta\omega_h}\right)^2}\frac{1}{1+\left[1+\left(2\frac{\omega_0-\omega}{\Delta\omega_h}\right)^2\right]^{-1}S/S_{sat}(\omega_0)} =$$

$$= \frac{\gamma_0(\omega_0)}{1+\frac{S}{S_{sat}(\omega_0)}+\left(2\frac{\omega_0-\omega}{\Delta\omega_h}\right)^2} = \frac{\gamma_0(\omega_0)}{1+\frac{S}{S_{sat}(\omega_0)}}\times\frac{1}{1+\left(2\frac{\omega_0-\omega}{\Delta\omega_h'(S)}\right)^2}$$

where the new power broadened linewidth is

$$\Delta\omega_h'(S) = \Delta\omega_h\sqrt{1+S/S_{sat}(\omega_0)}$$

Laser amplification

Now we can write the propagation equation for the signal intensity

$$\frac{d}{dz}S(z) = \gamma(S)S(z) = \frac{\gamma_0}{1+S(z)/S_{sat}}S(z)$$

In the absence of saturation, i.e. $S(z) << S_{sat}$ solution is

$$S_{out} = S(L) = S_{in}e^{\gamma_0 L} = G_0 S_{in}$$

G_0 is the small-signal or unsaturated gain.

Re-write the equation

$$\frac{1+S/S_{sat}}{S}dS=\gamma_0 dz$$

and integrate over the length of amplifier

$$\int_{S_{in}}^{S_{out}}\left[\frac{1}{S}+\frac{1}{S_{sat}}\right]dS=\gamma_0\int_0^L dz$$

Solution is

$$\ln\left(\frac{S_{out}}{S_{in}}\right)+\frac{S_{out}-S_{in}}{S_{sat}}=\gamma_0 L=\ln G_0$$

or

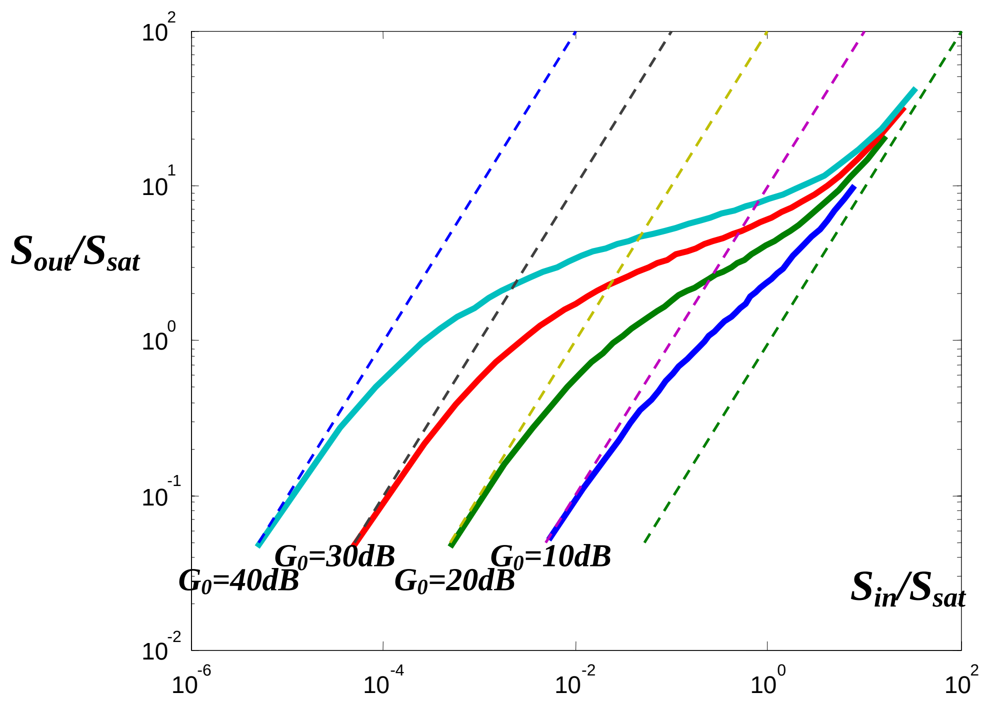
$$G\equiv\frac{S_{out}}{S_{in}}=G_0 e^{-\frac{S_{out}-S_{in}}{S_{sat}}}=G_0 e^{-(G-1)\frac{S_{in}}{S_{sat}}}=G_0 e^{-\frac{G-1}{G}\frac{S_{out}}{S_{sat}}}$$

We obtain a couple of equations in the parametric form

$$\frac{S_{in}}{S_{sat}}=\frac{1}{G-1}\ln\left(\frac{G_0}{G}\right);$$

$$\frac{S_{out}}{S_{sat}}=\frac{G}{G-1}\ln\left(\frac{G_0}{G}\right)$$

Note that for large enough S_{in}, G becomes close to unity.



Power extraction and available power (per unit area)

Power extraction is

$$S_{extr} \equiv S_{out} - S_{in} = \ln\left(\frac{G_0}{G}\right) \times S_{sat}$$

Maximum available power is

$$S_{avail} = \lim_{G\to 1} S_{extr} = \ln G_0 \times S_{sat} = \gamma_0 L \times S_{sat} = \sigma_{21} N_2^{(0)} L \times \frac{\hbar\omega}{\sigma_{21}\tau_2^{'}} = \frac{\hbar\omega}{\tau_2^{'}} N_2^{(0)} L$$

It all makes sense – the energy stored in unit volume is $\hbar\omega N_2^0$ and this energy can come out in one effective gain recovery time. Notice an important fact: according to our definition of the effective gain recovery one can extract a very large amount of power out of an amplifier even if the amount of energy stored in it is rather small. Consider an EDFA amplifier with the core area of about A~10^{-6}cm^2 , L=10 m length, Er ion concentration of .5x10^{19}cm^{-3} and upper level lifetime of τ_2 =10ms. The stored energy is

$$E_{st} = \hbar\omega NAL \approx 1.3\times 10^{-19} J \cdot 0.5\times 10^{19} cm^{-3} \cdot 10^{-3} cm^3 \approx 0.65 mJ$$

So, if one disregard shortening of effective gain recovery time it appears that available power is about 65mW – in reality one can get ten times that much by pumping hard. Indeed if we include the pump-caused shortening of the effective gain recover we obtain.

$$S_{avail} = \int_0^L \frac{\hbar\omega}{\tau_2^{'}} N_2^{(0)}(z)dz \approx \int_0^L \frac{\hbar\omega}{\tau_2^{'}} \frac{\sigma_{03} S_{pump}(z)}{\hbar\omega_{pump}} N\tau_2^{'} dz =$$

$$= \frac{\hbar\omega}{\hbar\omega_{pump}} S_{pump}(0) \int_0^L \sigma_{03} N e^{-\sigma_{13} N z} dz = \frac{\hbar\omega}{\hbar\omega_{pump}} S_{pump}(0)$$

Indeed, more than a Watt of power can be extracted from EDFA, limited only by availability of pump and optical damage.

Power extraction efficiency is

$$\eta_{extr} \equiv \frac{S_{extr}}{S_{avail}} \frac{\ln\left(\frac{G_0}{G}\right)}{\ln G_0} = 1 - G_{dB} / G_{0,dB}$$