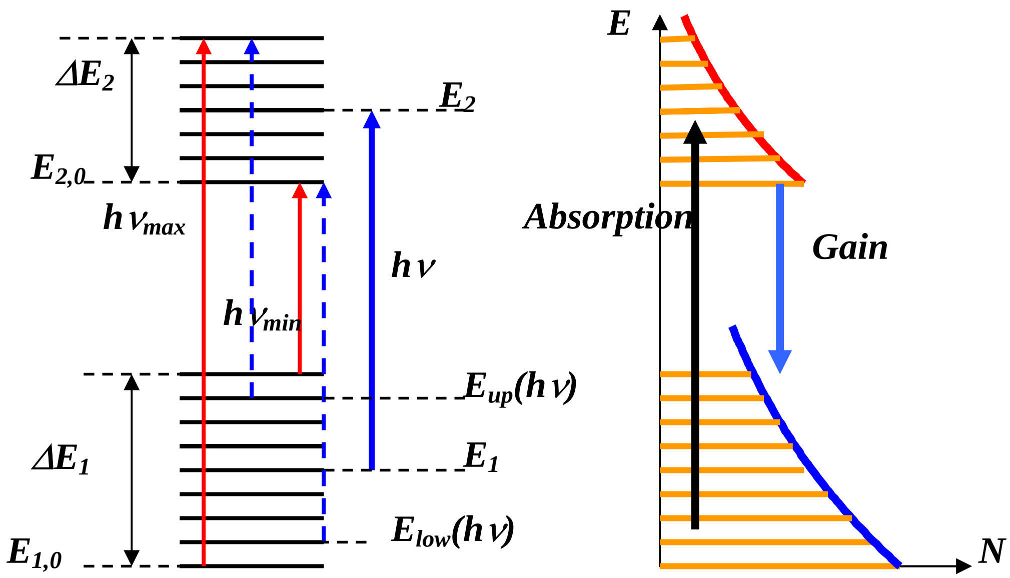
Lecture 6

Broadband gain

What if we have continuous distribution of states?



Let us say we have the lower states distributed over some manifold with energy spread ΔE_1 – and there is a "local thermal equilibrium" then we can write for the total population in the manifold 1

$$N_1 = N_1^0 \int_{E_{1,0}}^{E_{1,0}+\Delta E_1} \frac{g_1}{\Delta E_1} e^{-\frac{E_1 - E_{1,0}}{kT}} dE_1 = g_1 N_1^0 \frac{kT}{\Delta E_1}\left(1 - e^{-\frac{\Delta E_1}{kT}}\right)$$

Therefore, the population of atoms with energies in the range $E_1 \Rightarrow E_1 + dE_1$ is

$$dN_1(E_1) = \frac{g_1}{\Delta E_1} N_1^0 e^{-\frac{E_1 - E_{1,0}}{kT}} dE_1 = N_1 \frac{e^{-\frac{E_1 - E_{1,0}}{kT}}}{1 - e^{-\frac{\Delta E_1}{kT}}} \frac{dE_1}{kT}$$

Similarly, the population of atoms with energies in the range $E_2 \Rightarrow E_2 + dE_2$ is

$$dN_2(E_2) = \frac{g_2}{\Delta E_2} N_2^0 e^{-\frac{E_2 - E_{2,0}}{kT}} dE_2 = N_2 \frac{e^{-\frac{E_2 - E_{2,0}}{kT}}}{1 - e^{-\frac{\Delta E_2}{kT}}} \frac{dE_2}{kT}$$

The relation between the second Einstein coefficients for the transitions between these two intervals is

$$B_{12}(E_2, E_1) = \frac{g_2 / \Delta E_2}{g_1 / \Delta E_1} B_{21}(E_2, E_1)$$

Therefore, for the net population change in the presence of monochromatic light with energy spectral density

$$U_{\omega'} = U\delta(\omega' - \omega) = \frac{n}{c} S\delta(\omega' - \omega) = \frac{n}{c} S\hbar\delta(\hbar\omega' - \hbar\omega)$$

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} =$$

$$\frac{n}{c} S\hbar \int_{E_{10}}^{E_{10}+\Delta E_1} \int_{E_{20}}^{E_{20}+\Delta E_2} \left[B_{12}(E_1, E_2) dN_1(E_1) dE_2 - B_{21}(E_1, E_2) dN_2(E_2) dE_1 \right] \delta(E_2 - E_1 - \hbar\omega) =$$

$$= -\frac{n}{c} S\hbar \int_{E_{1,low}}^{E_{1,up}} B_{21}(E_1, E_1 + \hbar\omega) \left[dN_2(E_1 + \hbar\omega) - \frac{g_2 / \Delta E_2}{g_1 / \Delta E_1} dN_1(E_1) \right] =$$

$$= -\frac{n}{c} S \frac{\hbar}{kT} \int_{E_{1,low}}^{E_{1,up}} B_{21}(E_1, E_1 + \hbar\omega) \left[N_2 \frac{e^{-\frac{E_1 + \hbar\omega - E_{2,0}}{kT}}}{1 - e^{-\frac{\Delta E_2}{kT}}} - \frac{g_2 / \Delta E_2}{g_1 / \Delta E_1} N_1 \frac{e^{-\frac{E_1 - E_{1,0}}{kT}}}{1 - e^{-\frac{\Delta E_1}{kT}}} \right] dE_1 =$$

$$= -\frac{n}{c} S \frac{\hbar}{kT} \left[N_2 e^{-\frac{\hbar\omega - \left(E_{2,0} - E_{1,0}\right)}{kT}} - N_1 \frac{g_2 / \Delta E_2}{g_1 / \Delta E_1} \frac{1 - e^{-\frac{\Delta E_2}{kT}}}{1 - e^{-\frac{\Delta E_1}{kT}}} \right] \int_{E_{1,low}}^{E_{1,up}} B_{21}(E_1, E_1 + \hbar\omega) e^{-\frac{E_1 - E_{1,0}}{kT}} dE_1$$

On the other hand

$$\frac{dN_2}{dt} = -\frac{S}{\hbar\omega} \left[\sigma_{21}(\omega) N_2 - \sigma_{12}(\omega) N_1 \right]$$

Therefore

$$\sigma_{21}(\omega)=\frac{n}{c}\frac{\hbar^2\omega}{kT}e^{-\frac{\hbar\omega-\left(E_{2,0}-E_{1,0}\right)}{kT}}\int_{E_{1,low}}^{E_{1,up}}B_{21}(E_1,E_1+\hbar\omega)e^{-\frac{E_1-E_{1,0}}{kT}}dE_1$$

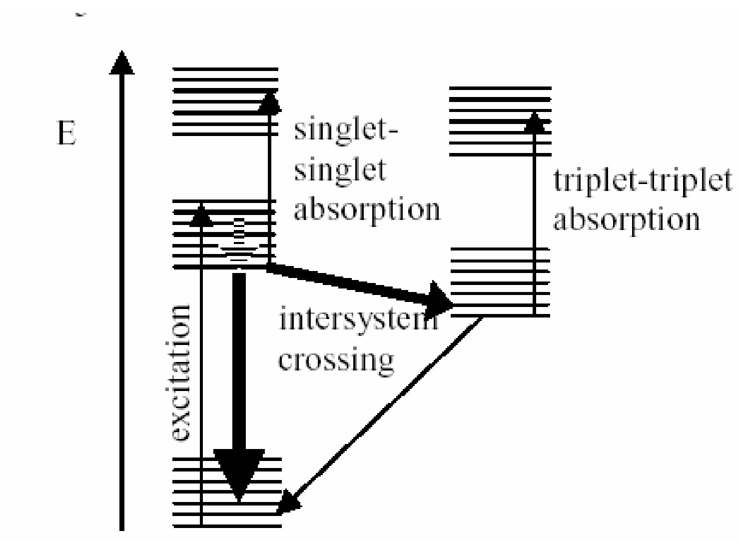
$$\sigma_{12}(\omega)=\sigma_{21}(\omega)\frac{g_2/\Delta E_2}{g_1/\Delta E_1}\frac{1-e^{-\frac{\Delta E_2}{kT}}}{1-e^{-\frac{\Delta E_1}{kT}}}e^{\frac{\hbar\omega-\left(E_{2,0}-E_{1,0}\right)}{kT}}=\sigma_{21}(\omega)e^{\frac{\hbar\omega-E_{ex}}{kT}}$$

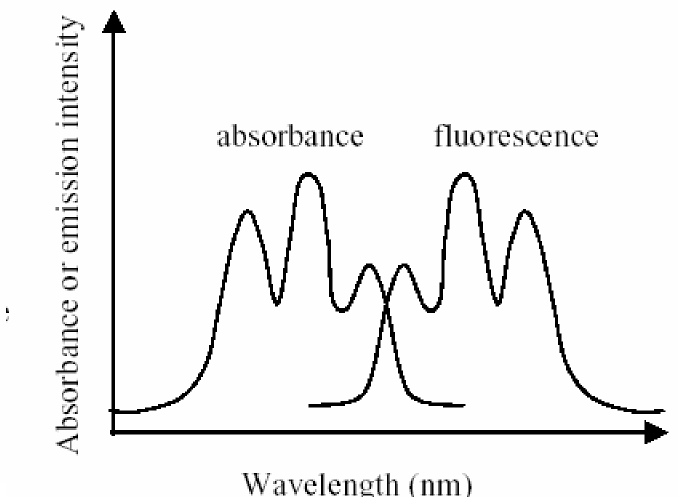
where E_{ex} is an excitation potential.

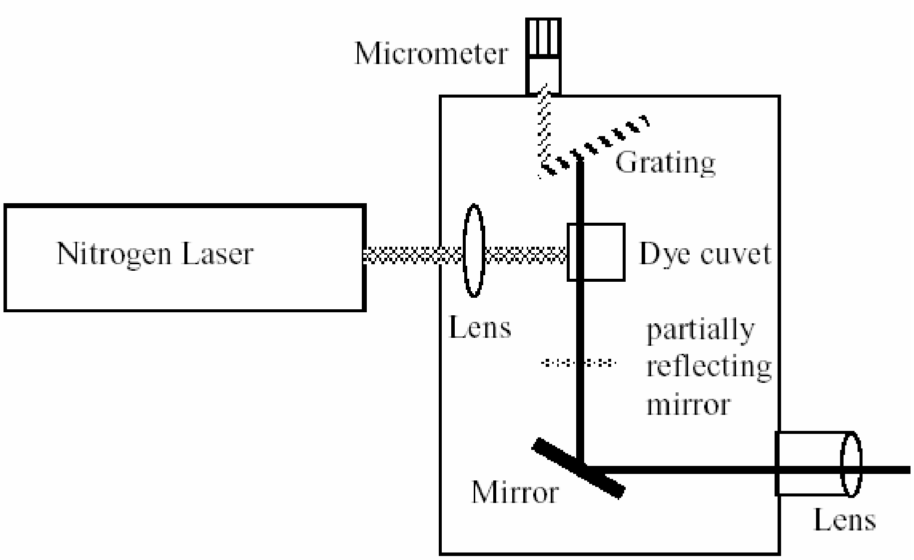
Thus we finally get for the gain $\gamma(\omega)=\sigma_{21}(\omega)[N_2-N_1e^{\frac{\hbar\omega-E_{ex}}{kT}}]$

Therefore we do not need global inversion, i.e. $N_2>N_1$ to obtain local gain.

Example: Dye laser







Example: EDFA