SOLUTION TO HW 3

The phase difference between the light coming from two adjacent slits is $\delta = dk\sin\theta$. The total number of slits is N. Total field is

$$E = E_0 \sum_{n=0}^{N-1} e^{jn\delta} = E_0 \frac{1-e^{j(N-1)\delta}}{1-e^{j\delta}}$$

$$E = E_0 \frac{e^{jN\delta/2}}{e^{j\delta/2}} \frac{e^{-jN\delta/2} - e^{jN\delta/2}}{e^{-j\delta/2} - e^{j\delta/2}} = Ee^{j(N-1)\delta/2} \frac{\sin\left(N\delta/2\right)}{\sin\left(\delta/2\right)}$$

$$S \sim |E|^2 = S_0 \frac{\sin^2\left(N\delta/2\right)}{\sin^2\left(\delta/2\right)}$$

$N = 3$

$$S = S_0 \left[\frac{\sin\left(3\delta/2\right)}{\sin\left(\delta/2\right)}\right]^2 = S_0 \left[\frac{\sin\left(\delta/2\right)\cos\delta + 2\sin\left(\delta/2\right)\cos^2\left(\delta/2\right)}{\sin\left(\delta/2\right)}\right]^2 = S_0\left[1+2\cos\delta\right]^2$$

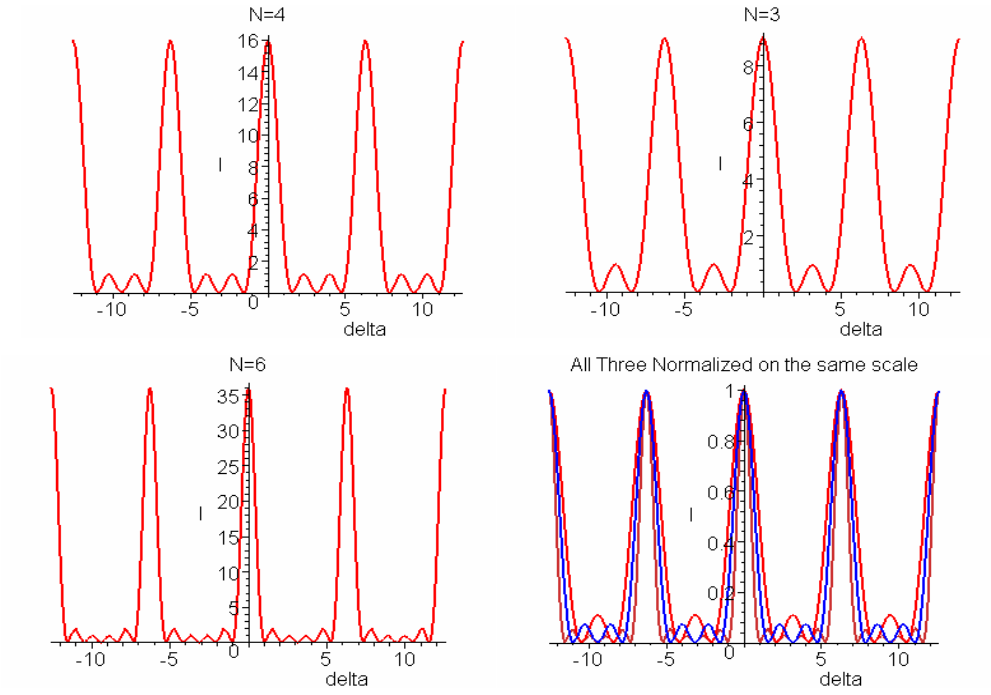
$N = 4$

$$S = S_0 \left[\frac{4\sin\left(\delta/2\right)\cos\left(\delta/2\right)\cos\delta}{\sin\left(\delta/2\right)}\right]^2 = 16S_0\cos^2\left(\delta/2\right)\cos^2\delta$$

$N = 6$

$$S = S_0 \frac{\sin^2\left(3\delta\right)}{\sin^2\left(\delta/2\right)}$$

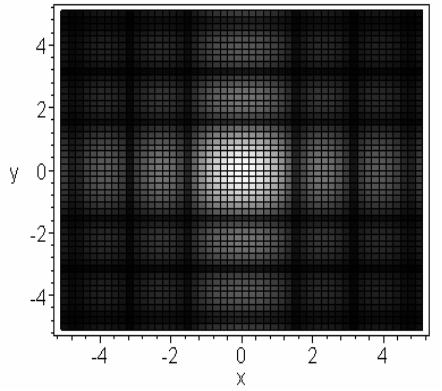
Here are the plots:

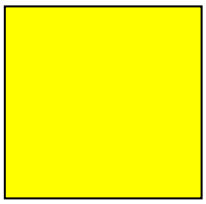


2. The diffraction pattern is a product of diffraction patterns along x and y co-ordiantes.

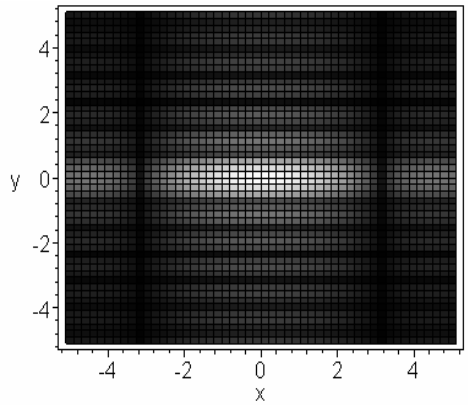
$$I(x,y)=\frac{\sin\ (kax/2L)}{(kax/2L)^2}\frac{\sin\ (kbx/2L)}{(kbx/2L)^2}$$

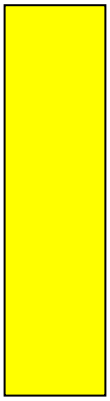
For a=b=2



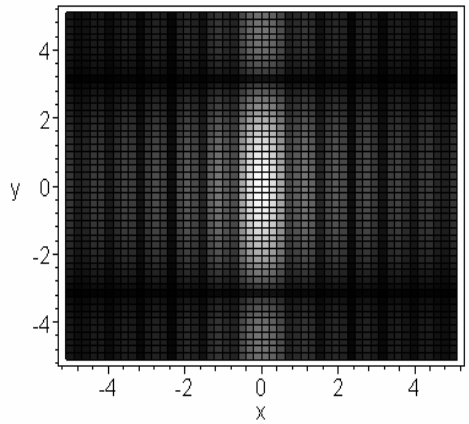


For a=1;b=4;





For a=4,b=1;



3. Radiation lifetime is $\tau_R = \dfrac{3m_0\lambda^2}{2\pi e^2\eta_0} \approx 45ns \times \lambda^2$ where λ is in μm

Thus for λ=1cm τ=45ns$\times 10^8 \approx$4.5s, for λ=0.5μm τ=45ns/4$\approx$11.2ns, for λ=1nm τ=45ns$\times 10^{-6} \approx$45fs!