2007 Midterm Solutions:

Problem 1:

Conductivity:
$\sigma=-e\mu_n N_n+e\mu_p N_p=e^2\tau_s(N_n/m_n+N_p/m_p)$

Hall effect

The drift current is the sum of electron and hole currents

$$\mathbf{J}_D=\mathbf{J}_{D,n}+\mathbf{J}_{D,p}=-eN_n\mathbf{v}_{D,n}+eN_n\mathbf{v}_{D,p}=e^2\tau_s\left(\frac{N_n}{m_n}+\frac{N_p}{m_p}\right)\mathbf{E}$$

$$\mathbf{J}_{D,n}=\mathbf{J}_D\frac{N_n/m_n}{N_n/m_n+N_p/m_p}\quad \mathbf{J}_{D,p}=\mathbf{J}_D\frac{N_p/m_p}{N_n/m_n+N_p/m_p}$$

Now, according to Eq. 30 (Chapter 1) the drift velocities of electrons and holes in the normal direction will be determined by the magnetic forces and the Hall field

$$\frac{m_n}{\tau_s}v_{D,n,y}=-e(E_H-Bv_{D,n,x})$$

$$\frac{m_p}{\tau_s}v_{D,n,y}=e(E_H-Bv_{D,p,x})$$

Individually the electron and hole currents in the normal direction do not have to be equal to zero. What should be equal to zero is the total current in the y direction,

$$J_y=-eN_nv_{D,n,y}+eN_pv_{D,p,y}=\tau_s[\frac{e^2N_n}{m_n}(E_H-Bv_{D,n,x})+\frac{e^2N_p}{m_p}(E_H-Bv_{D,p,x})]=$$

$$=\tau_s\left[e^2\left(\frac{N_n}{m_n}+\frac{N_p}{m_p}\right)E_H+eB\left(\frac{J_{D,n,x}}{m_n}-\frac{J_{D,p,x}}{m_p}\right)\right]=0$$

Then we can find the Hall field as

$$E_H=-\frac{B}{e}\frac{J_{D,n,x}/m_n-J_{D,p,x}/m_p}{N_n/m_n+N_p/m_p}=-\frac{J_{Dx}B}{e}\frac{N_n/m_n^2-N_p/m_p^2}{\left(N_n/m_n+N_p/m_p\right)^2}=R_HJ_{Dx}B$$

$$R_H=-\frac{N_n/m_n^2-N_p/m_p^2}{e\left(N_n/m_n+N_p/m_p\right)^2}$$

Clearly one can have R_H=0!

Cyclotron resonance

There will be two separate resonances with frequencies $\omega_{cn}=|e|/m_nB$ and $\omega_{cp}=|e|/m_pB$

Thermo-electric effect

The thermo-electric current is:

$$\mathbf{J}_{te} = \mathbf{J}_{te,n} + \mathbf{J}_{te,p} = \frac{1}{6} eN_n\tau_s \frac{dv_{th,n}^2}{dT}\nabla T - \frac{1}{6} eN_p\tau_s \frac{dv_{th,p}^2}{dT}\nabla T = \frac{1}{6} e\tau_s\nabla T\left(N_n \frac{dv_{th,n}^2}{dT} - N_p \frac{dv_{th,p}^2}{dT}\right)$$

The drift current that compensates thermo-electric current is

$$\mathbf{J}_D = \mathbf{J}_{D,n} + \mathbf{J}_{D,p} = e^2\tau_s\left(\frac{N_n}{m_n} + \frac{N_p}{m_p}\right)\mathbf{E}_{te} = -\mathbf{J}_{te}$$

Therefore

$$\mathbf{E}_{te} = -\frac{1}{6e}\nabla T\left(\frac{N_n}{m_n}\frac{dm_nv_{th,n}^2}{dT} - \frac{N_p}{m_p}\frac{dm_pv_{th,p}^2}{dT}\right)\left(\frac{N_n}{m_n} + \frac{N_p}{m_p}\right)^{-1} = -\frac{k_B}{2e}\frac{N_nm_p - N_pm_n}{N_nm_p + N_pm_n}\nabla T$$

$$Q = -\frac{k_B}{2e}\frac{N_nm_p - N_pm_n}{N_nm_p + N_pm_n}$$

Once again, there can be complete compensation.

Problem 2:

The time-dependent force Lorentz that pushes the atom is

$$F_M(t) = ev(t)B(t) = \frac{en}{c}v(t)E(t)$$

and $E(t) = E_\omega \cos\omega t = \mathrm{Re}(E_\omega e^{j\omega t})$ (where we use scalar notation). From the resonant dipole model of atom we obtain for the velocity

$$v(t) = \mathrm{Re}(v_\omega e^{j\omega t}) = \mathrm{Re}\left[\frac{\partial}{\partial t}(r_\omega e^{-j\omega t})\right] = \mathrm{Re}\left[-j\omega r_\omega e^{-j\omega t}\right] = \omega\,\mathrm{Im}\left(r_\omega e^{-j\omega t}\right)$$

Where r is the displacement of electron. Now we can use the fact that atom displacement is related to polarization as $P = eNr,$ and $P = \chi\varepsilon_0 E$ to express the displacement as

$$r_\omega = (eN)^{-1}P_\omega = \left(eN\right)^{-1}\varepsilon_0\chi_\omega E_\omega$$

Thus

$$v(t) = \left(eN\right)^{-1}\varepsilon_0\omega\,\mathrm{Im}\left(\chi_\omega E_\omega e^{-j\omega t}\right) = \left(eN\right)^{-1}\varepsilon_0 E_\omega\omega\,\mathrm{Im}\left[\left(\chi_\omega^{'} + j\chi_\omega^{''}\right)(\cos\omega t + j\sin\omega t)\right] =$$
$$= \left(eN\right)^{-1}\varepsilon_0 E_\omega\omega\left[\chi_\omega^{''}\cos\omega t + \chi_\omega^{'}\sin\omega t\right]$$

Therefore

$$F_M(t) = enc^{-1}\left(eN\right)^{-1}\varepsilon_0 E_\omega\omega\left[\chi_\omega^{''}\cos\omega t + \chi_\omega^{'}\sin\omega t\right]E_\omega\cos\omega t$$

Note that the electron charge dependence is gone now - thus the force does not really depend on the medium model. Average over time

$$\langle F_M \rangle = n\varepsilon_0 \frac{\omega}{c} N^{-1} E_\omega^2 \left[\chi_\omega^{"} \langle \cos^2 \omega t \rangle + \chi_\omega^{'} \langle \sin \omega t \cos \omega t \rangle \right] = N^{-1} \frac{\omega}{nc} \chi_\omega^{"} \varepsilon_0 n^2 \frac{E_\omega^2}{2} =$$

Now use the relation between the absorption coefficient α_ω and χ_ω'' the notes (3.47)

$$\alpha_\omega = \frac{\omega}{nc} \chi_\omega^{"}$$

and the definitions of the average energy density

$$\langle U_\omega \rangle = \varepsilon_0 n^2 \frac{E_\omega^2}{2}$$

and time –averaged Poynting vector

$$\langle S_\omega \rangle = \frac{c}{n} \langle U_\omega \rangle$$

To obtain

$$\langle F_M \rangle = N^{-1} \alpha_\omega \langle U_\omega \rangle = \frac{n}{c} N^{-1} \alpha_\omega \langle S_\omega \rangle = \frac{n}{c} P_{abs}$$

where P_{abs} is the power per atom absorbed from the light.