

# MOS Transistors Models

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# The MOS transistor – Levels of Abstraction-

## Model Equations

$$\text{If } V_{GS} < 0 \\ I_{DS} = 0$$

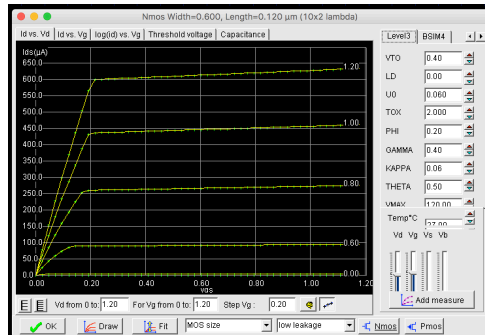
$$\text{If } V_{DS} > V_{GS} - V_{TO}$$

$$I_{DS} = \left(\frac{W}{L}\right) \left(\frac{UO}{2}\right) \frac{\epsilon_0 \epsilon_r}{TOX} (V_{GS} - V_T)^2$$

$$\text{If } V_{DS} < V_{GS} - V_{TO}$$

$$I_{DS} = \left(\frac{W}{L}\right) UO \frac{\epsilon_0 \epsilon_r}{TOX} \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

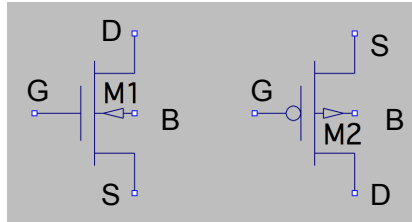
$$V_T = V_{TO} + \gamma (\sqrt{\phi - V_{BS}} - \sqrt{\phi})$$



Model Current-Voltage Characteristics

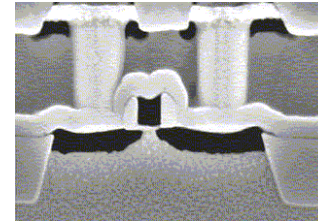
**MATHEMATICAL**

## Symbol

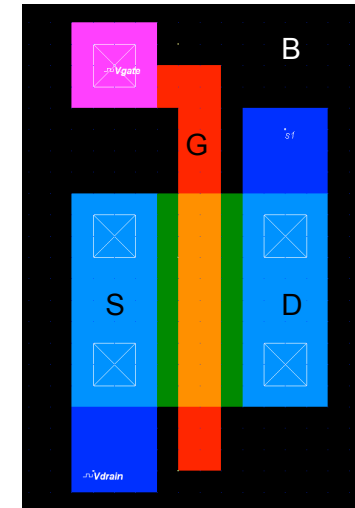


NMOS

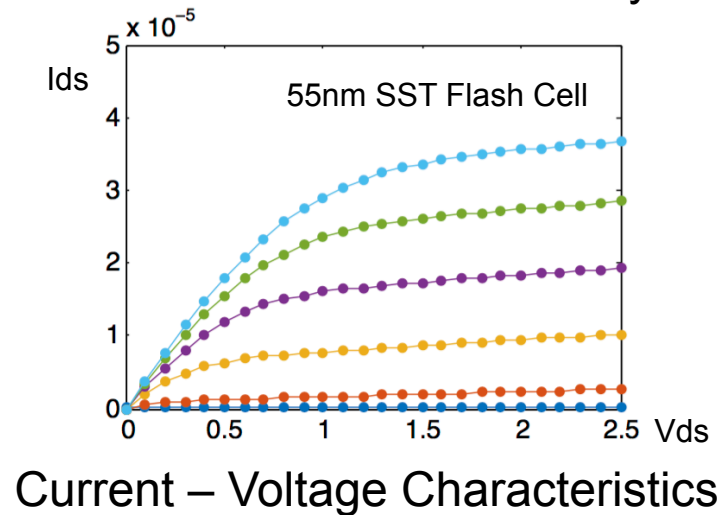
PMOS



SEM photograph



Layout



Current – Voltage Characteristics

**PHYSICAL**

## What is a model?

1. An intuitive and conceptual abstraction of a complex physical process
2. A mathematical abstraction of a complex physical process that is capable of predicting experimental observations.

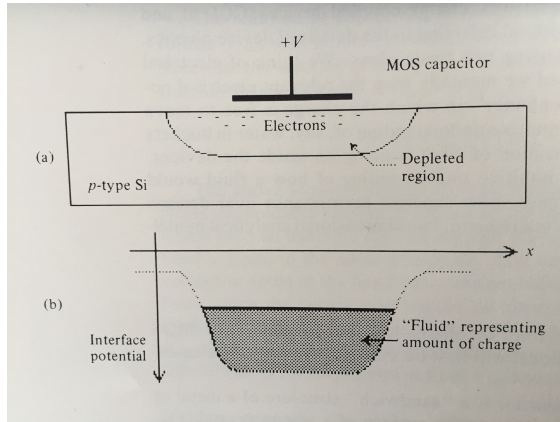
## What is a MODEL?

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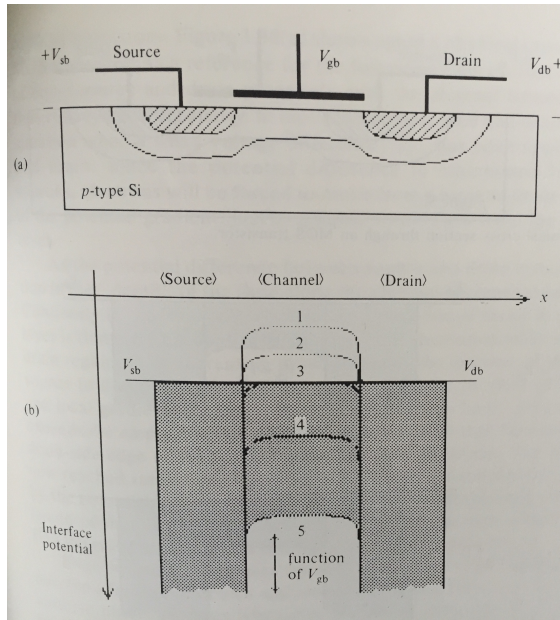
# MOS fluidic analogy: a conceptual model (I)

From Mead and Conway

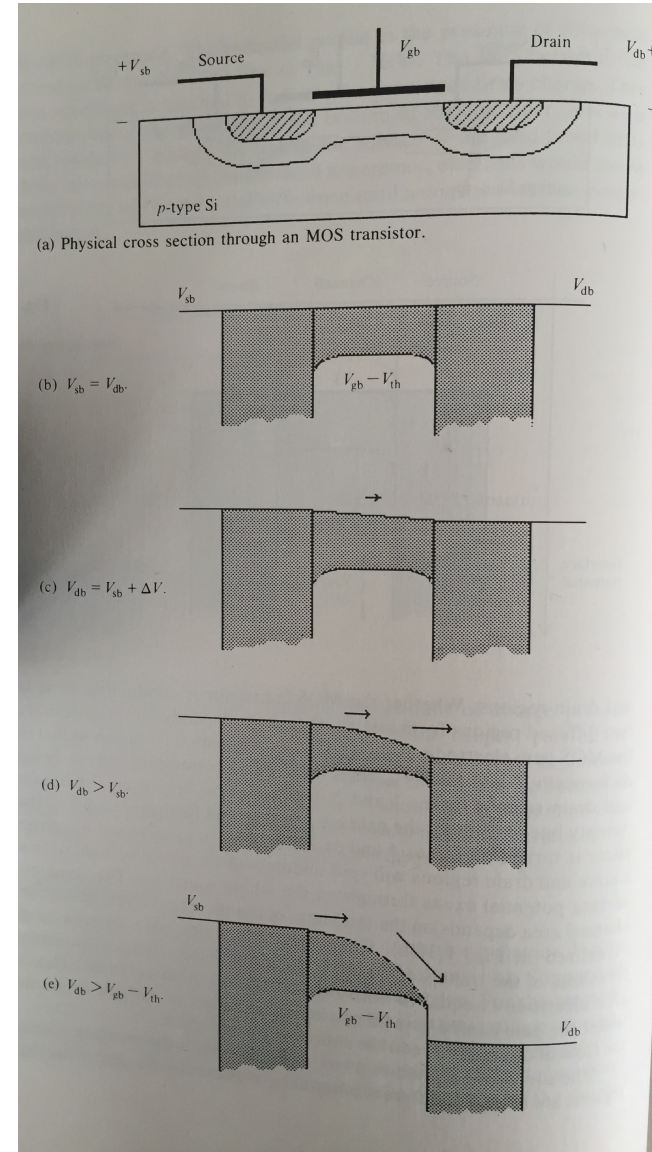
## MOS Capacitor



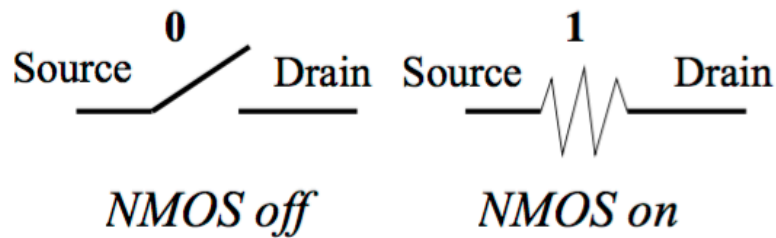
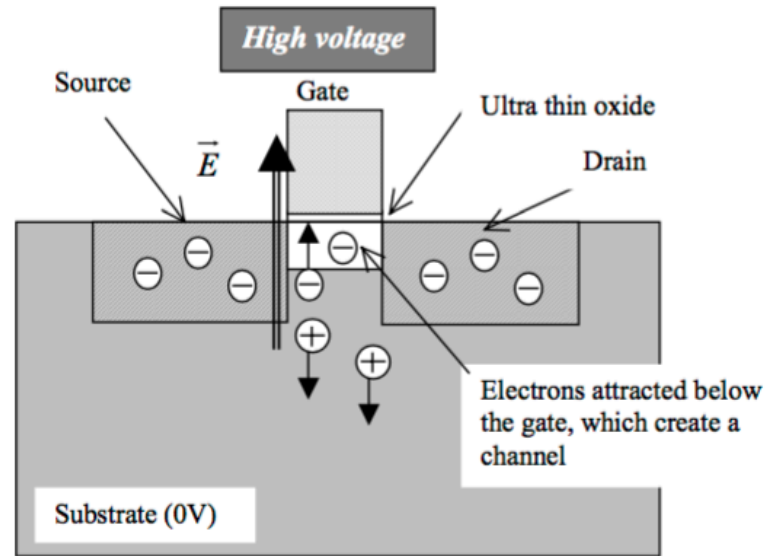
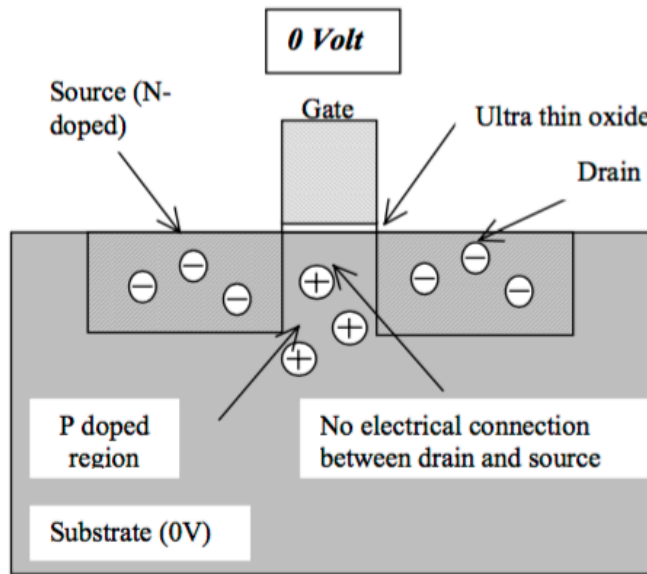
## MOS Transistor



## MOS Transistor Variable Bias



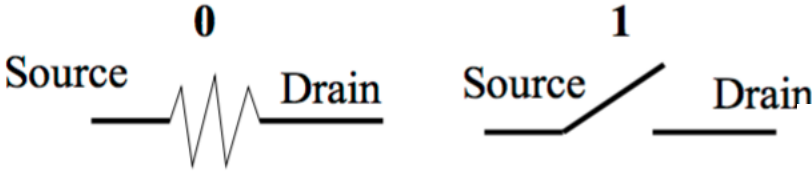
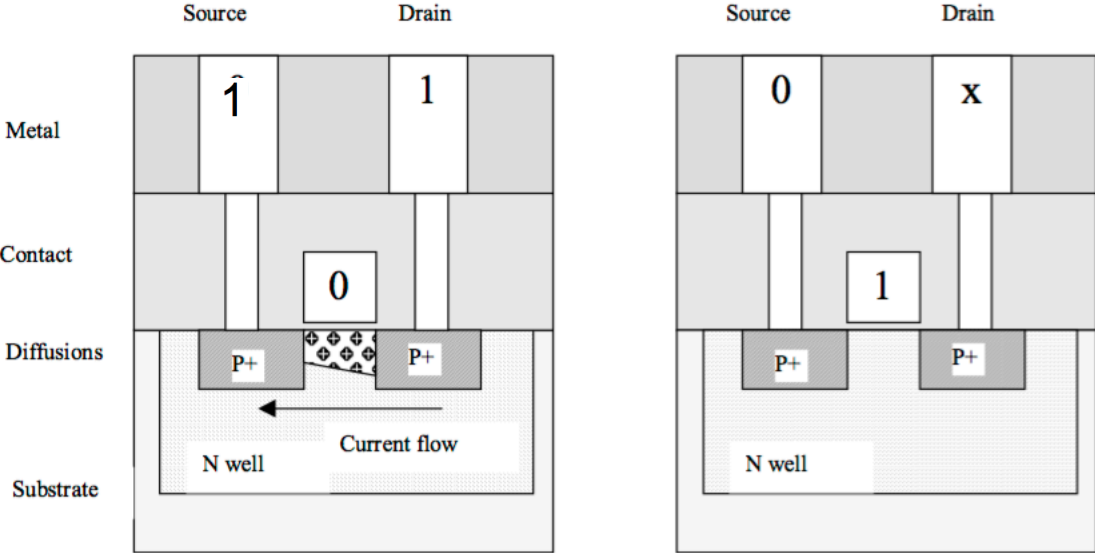
# NMOS as a switch/resistor: a conceptual model (II)



Gate	Source	Drain
0	0	X
0	1	X
1	0	0
1	1	1

not a good one

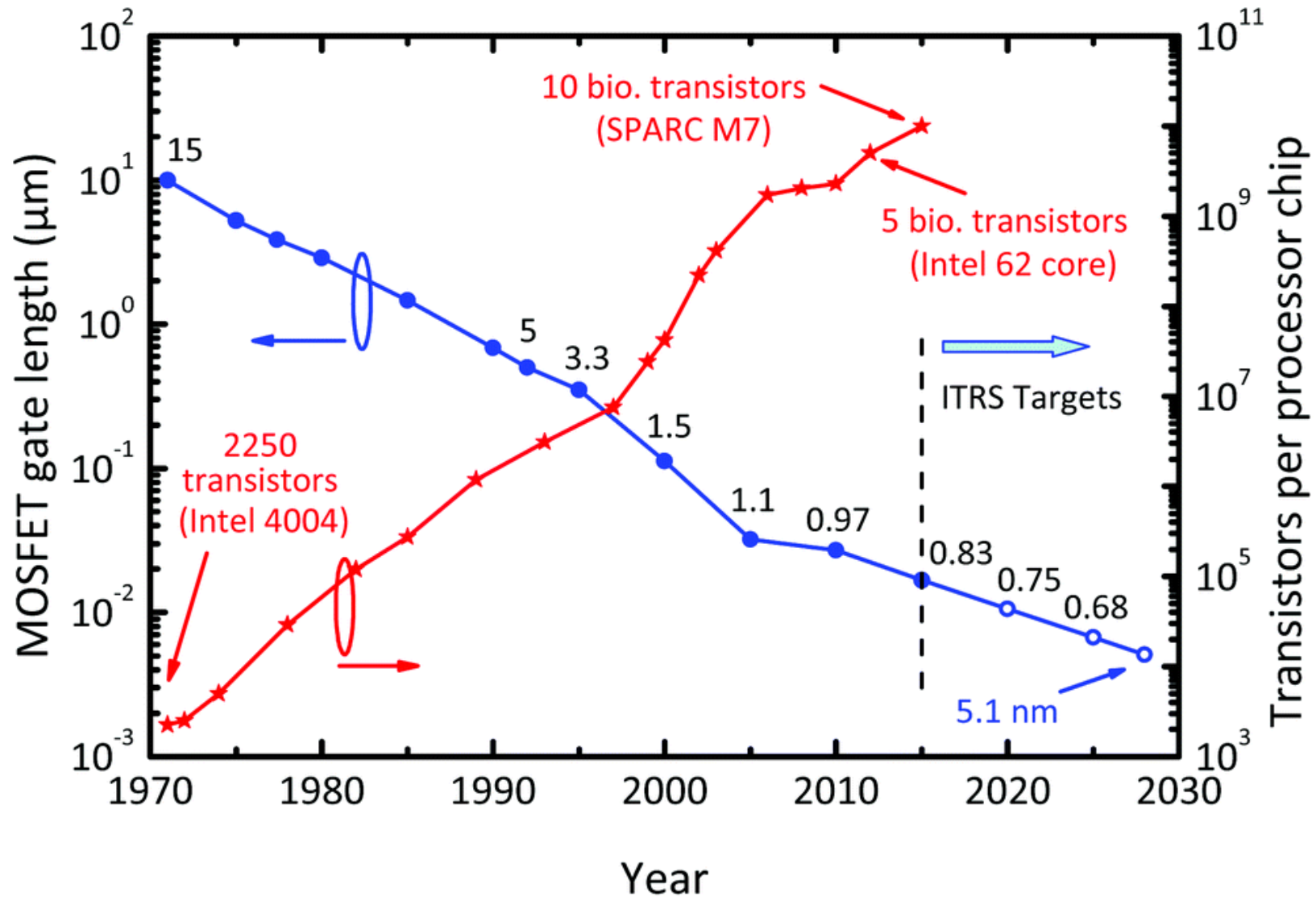
# PMOS as a switch/resistor: a conceptual model (III)



not a good zero

Gate	Source	Drain
0	0	0
0	1	1
1	0	X
1	1	X

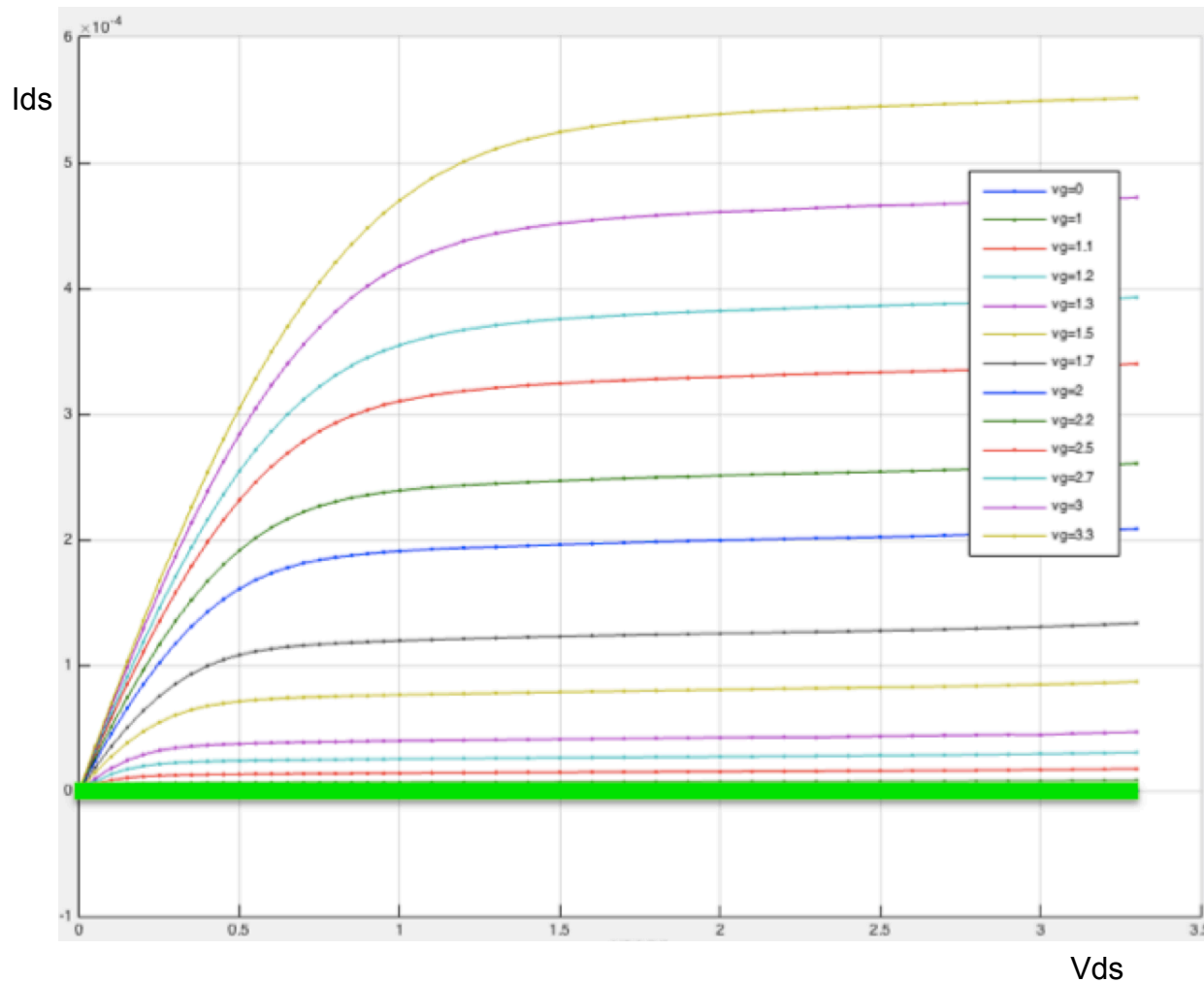
# What are the physical values for 0 and 1



chip/logic voltages (blue line)



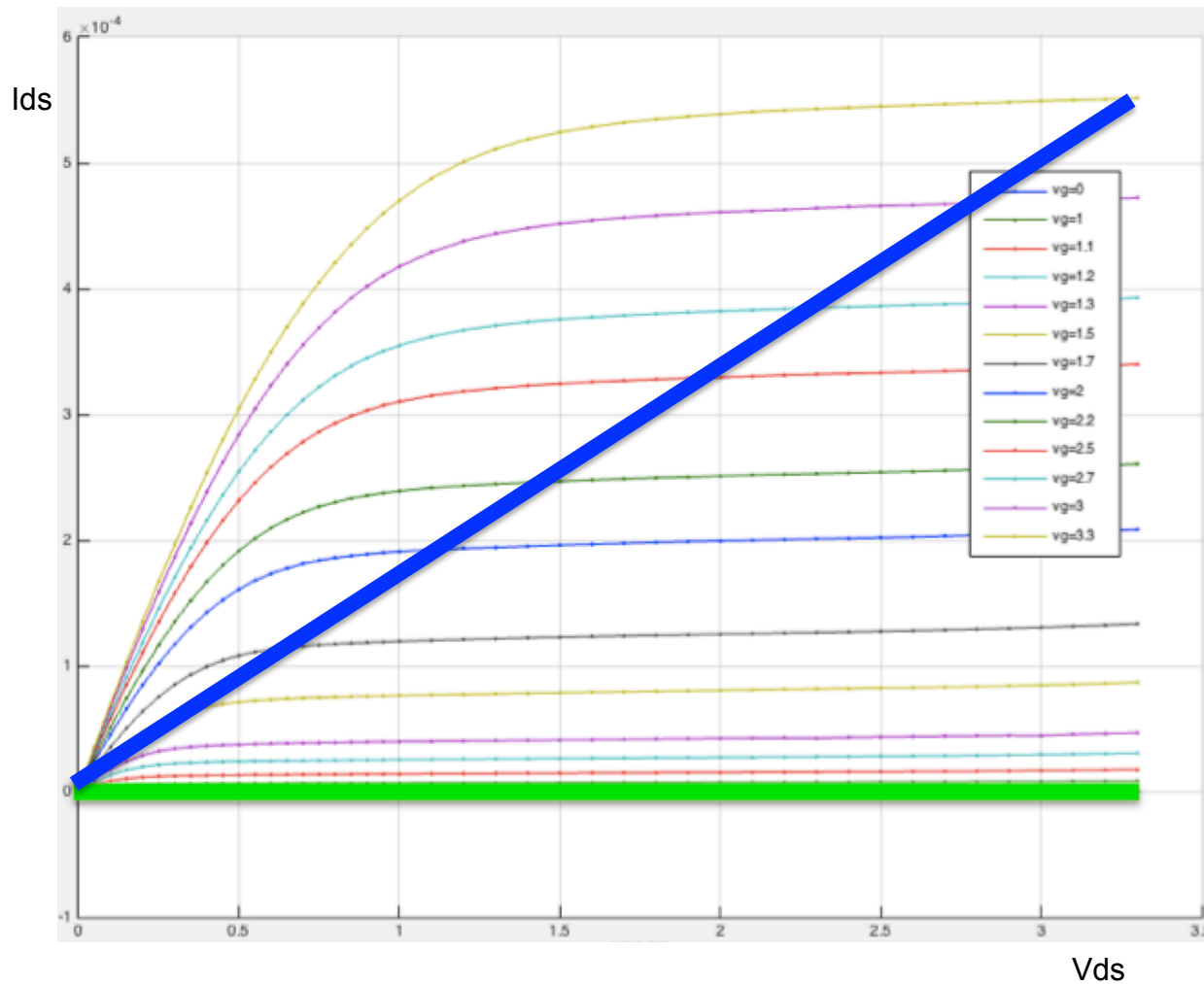
# MOS switch model relation to I-V characteristics (I)



180nm technology

With digital input on gate the device is either **ON** or **OFF**

## MOS switch model relation to I-V characteristics (II)



180nm technology

With digital input on gate the device is either **ON** or **OFF**

Approximate **ON** with the blue line

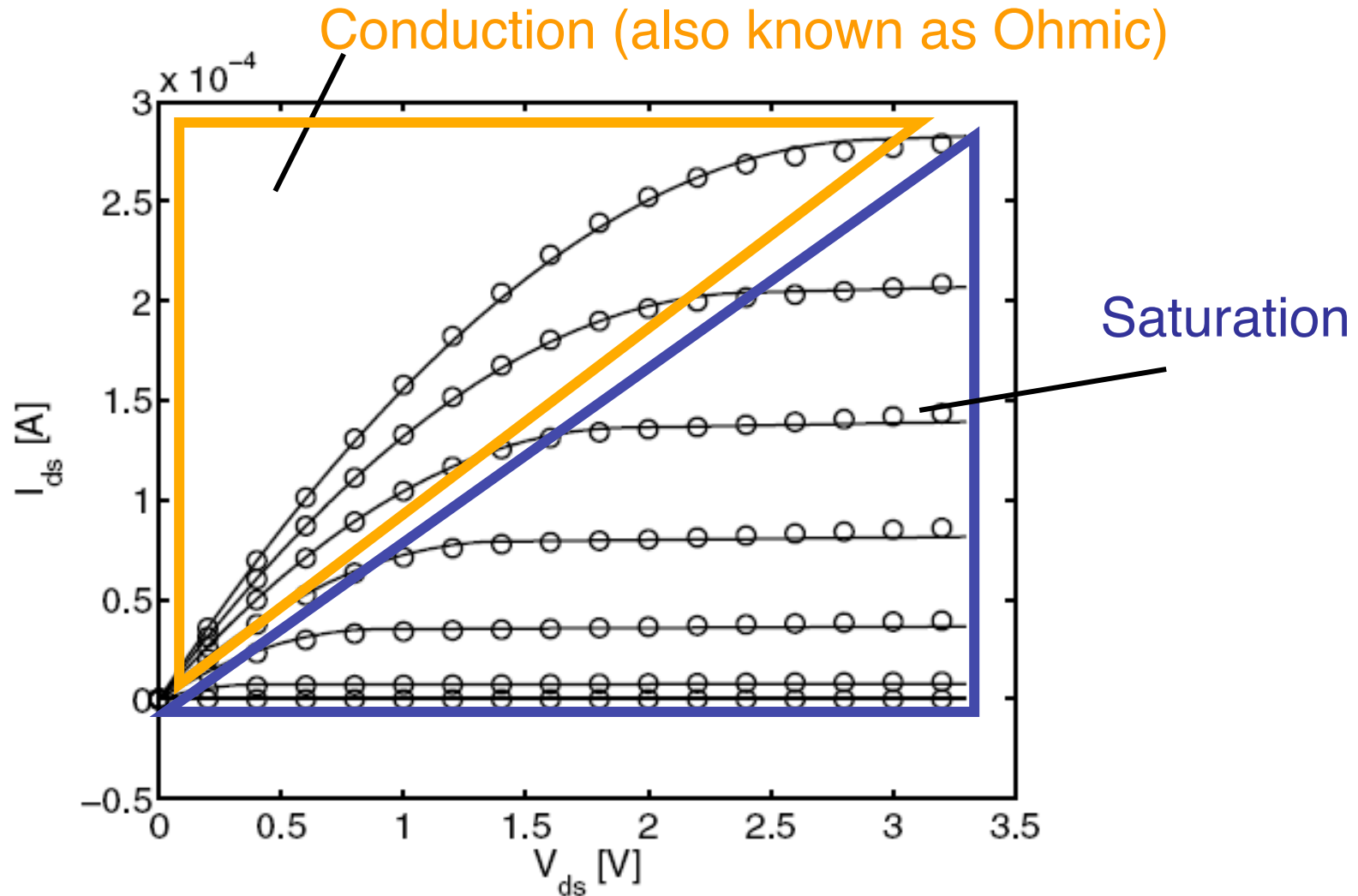
$$R_{ON} \sim 3.3 \text{ V} / 0.55 \text{ mA} = 6\text{K}$$

## What is a MODEL?

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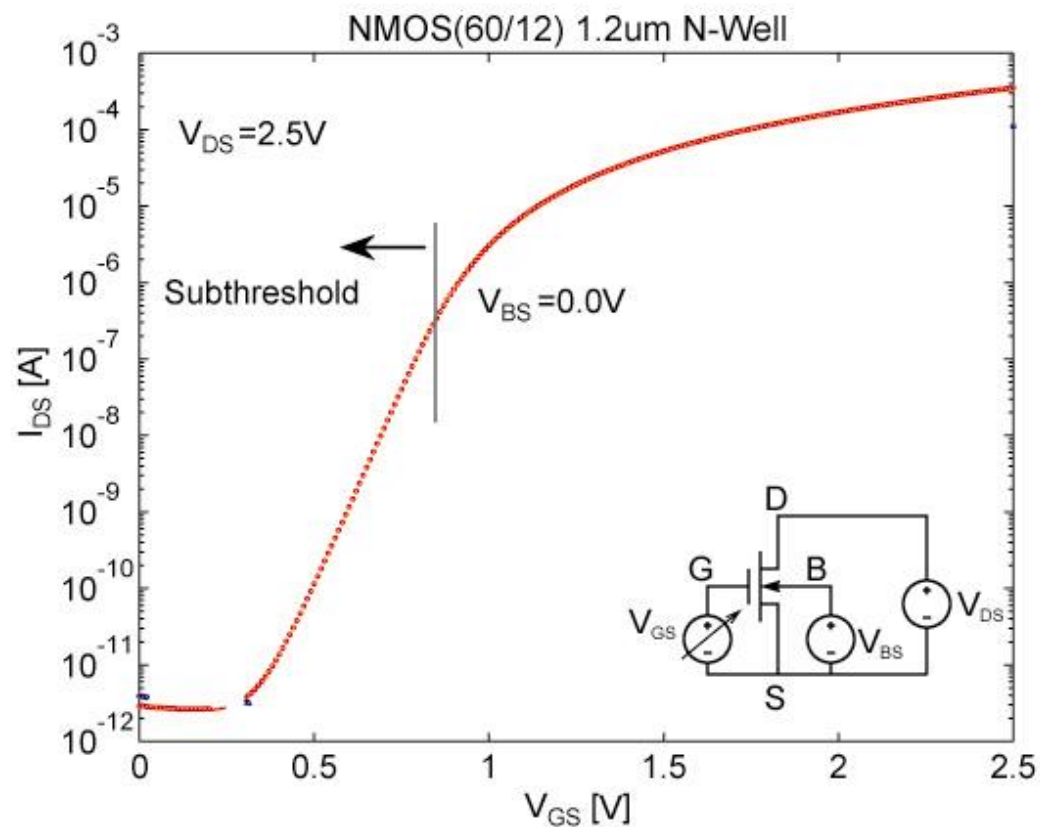
## Conduction –ohmic- vs saturation

Device operation characterized by the form of the current as a function of the bias voltage between the DRAIN and the source terminals ( $V_{ds}$ )

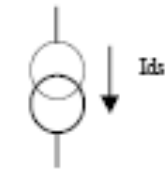
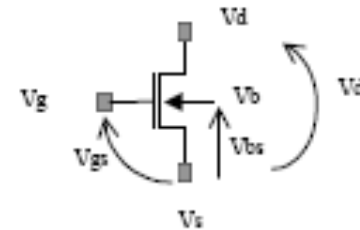
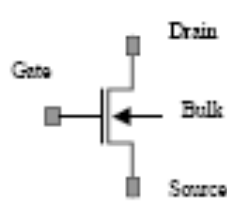
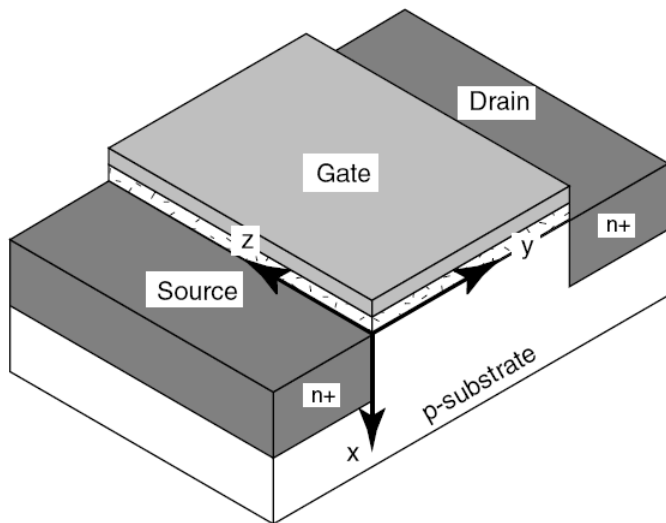


## Above threshold vs sub-threshold behaviour

Device operation characterized by the form of the current as a function of the bias voltage between the gate and the source terminals ( $V_{GS}$ )



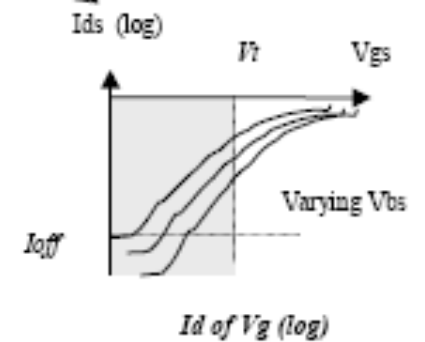
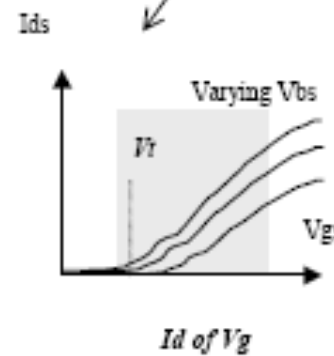
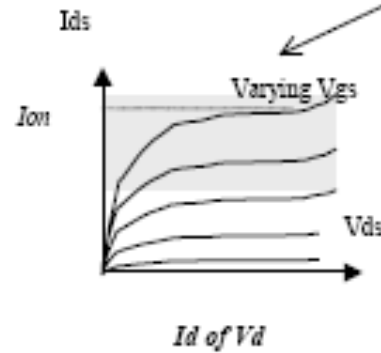
# MOS transistor mathematical model



$$I_{ds} = f(V_d, V_g, V_s, V_b)$$

(eq 3-1)

$$I_{ds} = f(v_d, v_s, v_g, v_b)$$



# Mathematical model –above threshold-

$$\text{If } V_{GS} < 0$$

$$I_{DS} = 0$$

$$\text{If } V_{DS} > V_{GS} - V_{TO}$$

Saturation

$$I_{DS} = \left(\frac{W}{L}\right) \left(\frac{UO}{2}\right) \frac{\epsilon_0 \epsilon_r}{TOX} (V_{GS} - V_T)^2$$

$$\text{If } V_{DS} < V_{GS} - V_{TO}$$

Ohmic

$$I_{DS} = \left(\frac{W}{L}\right) UO \frac{\epsilon_0 \epsilon_r}{TOX} \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

$\epsilon_0 = 8.85 \cdot 10^{-12}$  F/m is the absolute permittivity

$\epsilon_r$  = relative permittivity, equal to 3.9 in the case of SiO2 (no unit)

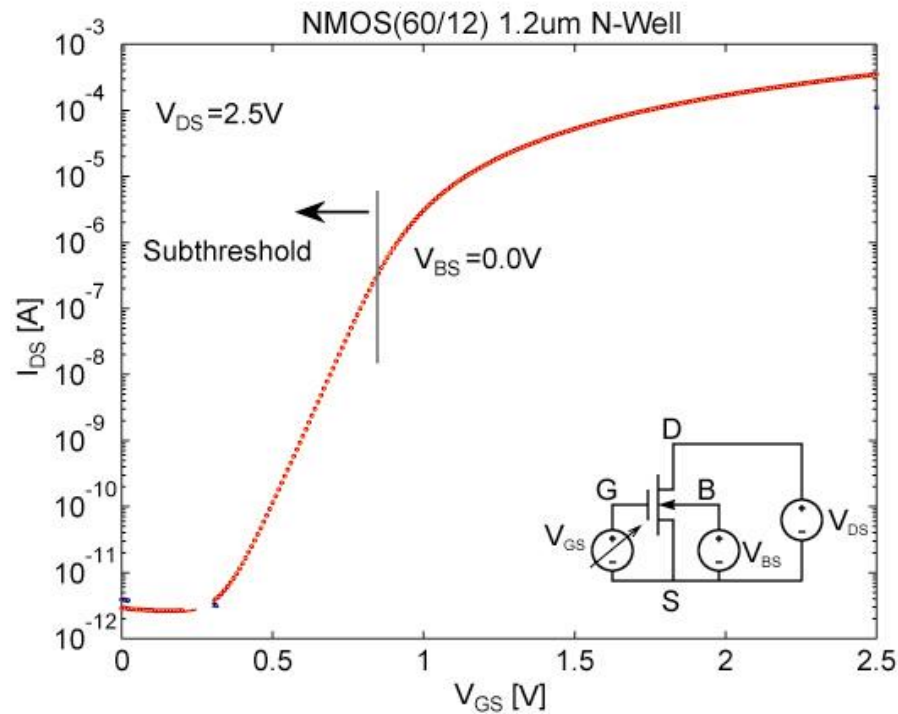
$$V_T = V_{TO} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

Mos Model 1 parameters			
Parameter	Definition	Typical Value 0.12µm	
		NMOS	PMOS
VTO	Theshold voltage	0.4V	-0.4V
U0	Carrier mobility	0.06m <sup>2</sup> /V-s	0.02m <sup>2</sup> /V-s
TOX	Gate oxide thickness	2nm	2nm
PHI	Surface potential at strong inversion	0.3V	0.3V
GAMMA	Bulk threshold parameter	0.4 V <sup>0.5</sup>	0.4 V <sup>0.5</sup>
W	MOS channel width	1µm	1µm
L	MOS channel length	0.12µm	0.12µm

Model parameters

## Above threshold vs sub-threshold behaviour

Device operation characterized by the form of the current as a function of the bias voltage between the gate and the source terminals ( $V_{GS}$ )





## Mathematical model –subthreshold-

$$I_D \equiv I_{DS} = S I_{n0} \exp\left(\frac{\kappa_n V_{GB}}{V_t}\right) \left[ \exp\left(\frac{-V_{SB}}{V_t}\right) - \exp\left(\frac{-V_{DB}}{V_t}\right) \right]$$

$$I_D \equiv I_{SD} = S I_{p0} \exp\left(\frac{-\kappa_p V_{GB}}{V_t}\right) \left[ \exp\left(\frac{V_{SB}}{V_t}\right) - \exp\left(\frac{V_{DB}}{V_t}\right) \right]$$

$$V_t \equiv \frac{kT}{q} \quad \kappa \equiv \frac{1}{\eta} \equiv \frac{C_{ox}}{C_{ox} + C_{dep}} \quad S \equiv \frac{W}{L}$$

$$I_{p0} = 0.5 \times 10^{-18} \text{ A}$$

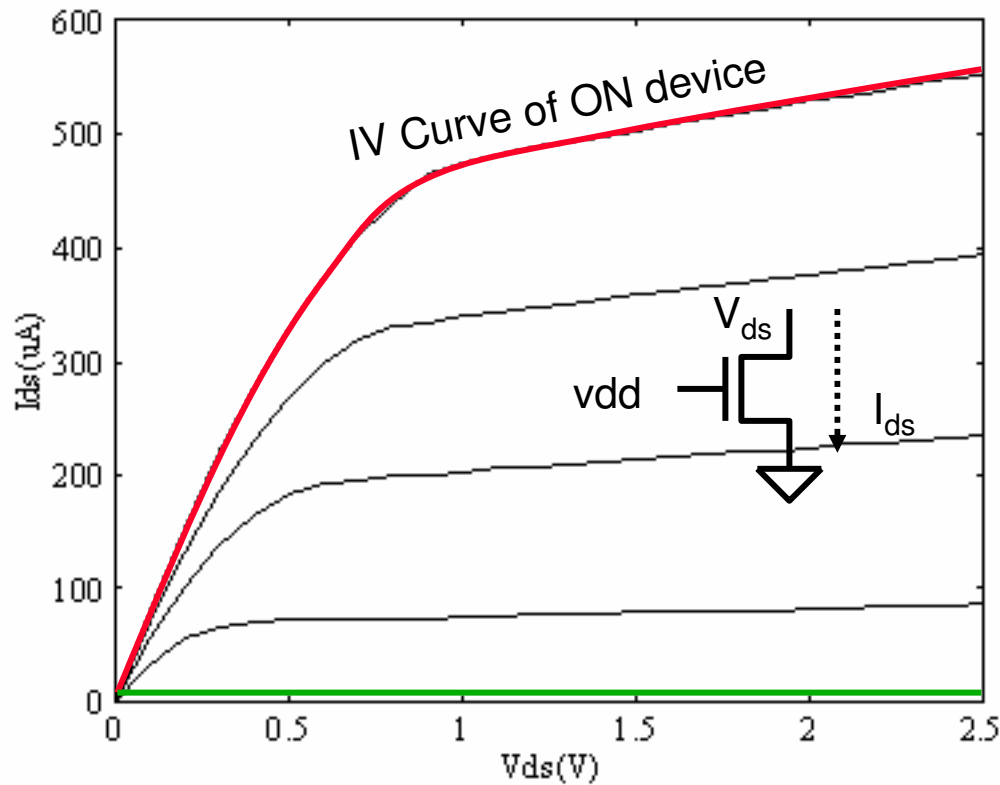
$$I_{n0} = 0.9 \times 10^{-18} \text{ A}$$

$$\kappa = 0.7$$

$$V_t = 0.26 \text{ Volts}$$

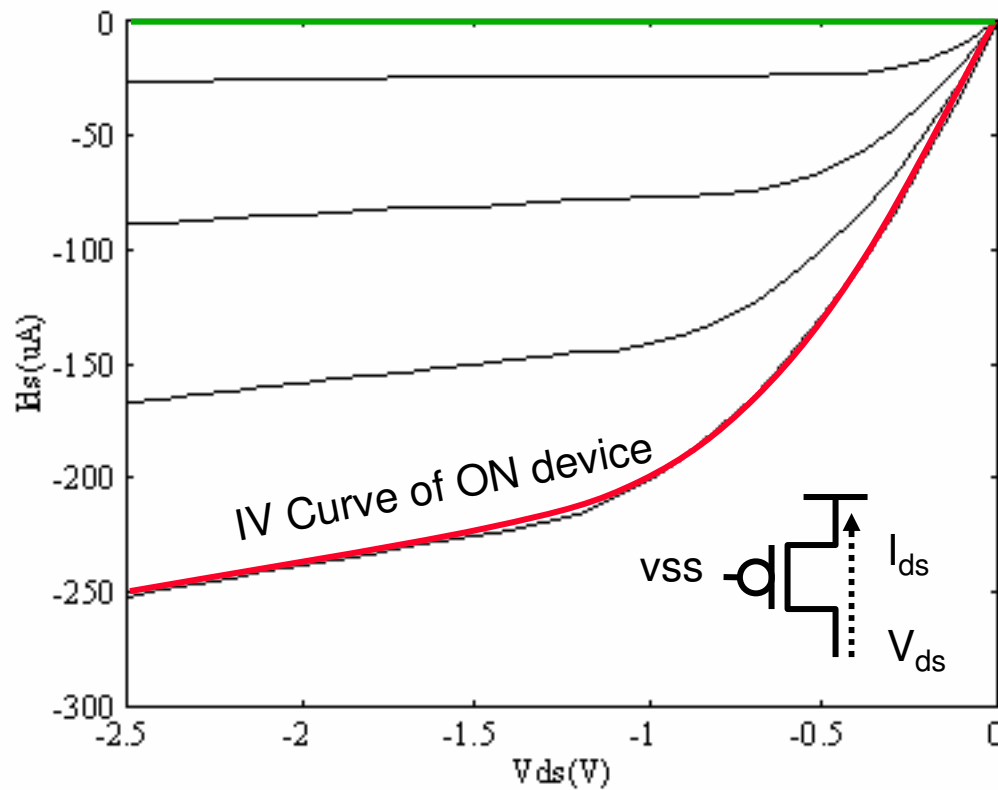
Parameters:

# Operating current for an NMOS



- Operates on one of two curves
- on
  - Looks like a current source initially (high  $V_{ds}$ )
  - Looks like a resistor later (low  $V_{ds}$ )
- off
  - Open circuit always

# Operating current for a PMOS



- Same behavior as NMOS
  - Open circuit when off
  - Current source or resistor when on

# Computer Aided Design Tools

