Chapter 1

Switch-based logic functions

1.1 Basic flashlight

A schematic is a diagram showing the important electrical components of an electrical circuit and their interconnections. One of the simplest electrical circuits that we can draw is the schematic diagram for an ordinary flashlight.

To draw a schematic for a flashlight, we need three basic elements: a voltage source (which represents the battery), a light bulb and a switch. These elements are shown in Figure 1.1, with the switch shown in its open (off) and closed (on) configurations.


(a) (b) (c) (d)

Figure 1.1: Schematic elements. (a) Voltage source. (b) Light bulb. (c) Switch, open (off). (d) Switch, closed (on).

Using these components, we can draw the schematic for the flashlight, with the switch in the on (or closed) position (Figure 1.2(a)) and with the switch in the off (or open) position (Figure 1.2(b)).

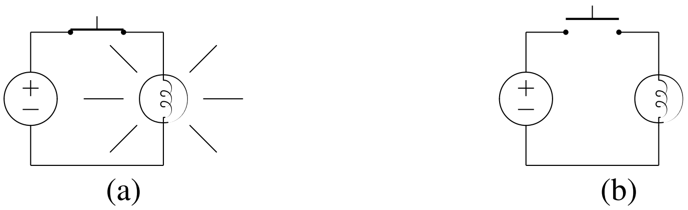

(a) (b)

Figure 1.2: Flashlight schematic. (a) Switch on. (b) Switch off.

The light bulb only turns on when there is a continuous, unbroken path from the voltage source to the light bulb and back again.

1.2 Boolean AND function

We can reproduce the boolean AND function by incorporating two switches into our circuit, as shown in the diagrams of Figure 1.3. Since there are two switches, and each switch can be in one of two states (either open or closed), there are a total four (2^2) possible states for the circuit. These four states are all shown in Figure 1.3.

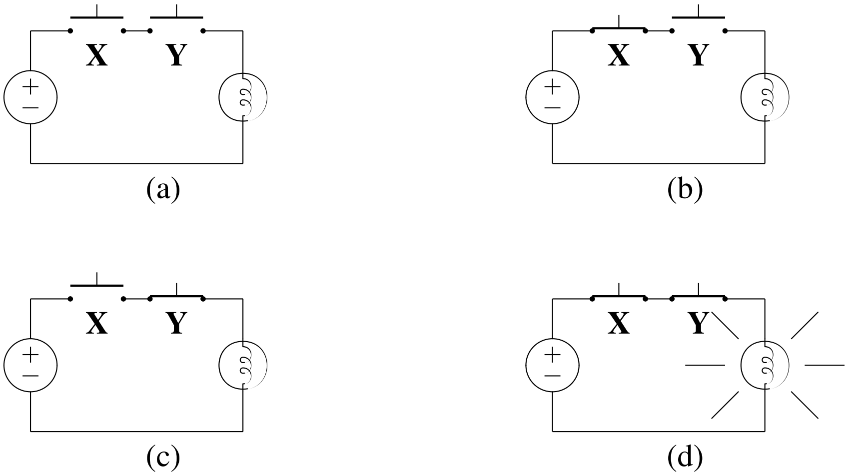


Figure 1.3: Two switch flashlight schematic. (a) Switches **X** and **Y** off. (b) Switches **X** on and **Y** off. (c) Switches **X** off and **Y** on. (d) Switches **X** and **Y** on.

By following the path from the voltage source to the light bulb and back again, we find that there is only one case in which this path is unbroken: for the light bulb to turn on, switches **X** AND **Y** must both be on.

Also note the manner in which the switches are connected. The path from the voltage source to the light bulb passes through each switch serially. This arrangement of components is called a *series* connection.

1.3 Boolean OR function

We can also reproduce the boolean OR function with two switches as shown in the diagrams of Figure 1.4. Again, there are two switches and each switch can be in one of two states, so there are a total four (2^2) possible states for the circuit. These four states are all shown in Figure 1.4.

By following the path from the voltage source to the light bulb and back again, we find that there are three case in which this path is unbroken: for the light bulb to turn on, switches **X** OR **Y** must be on.

This circuit differs from the previous one only in the manner in which the switches are connected. The path from the voltage source to the light bulb can pass through either switch or both simultaneously. This arrangement of components is called a *parallel* connection.

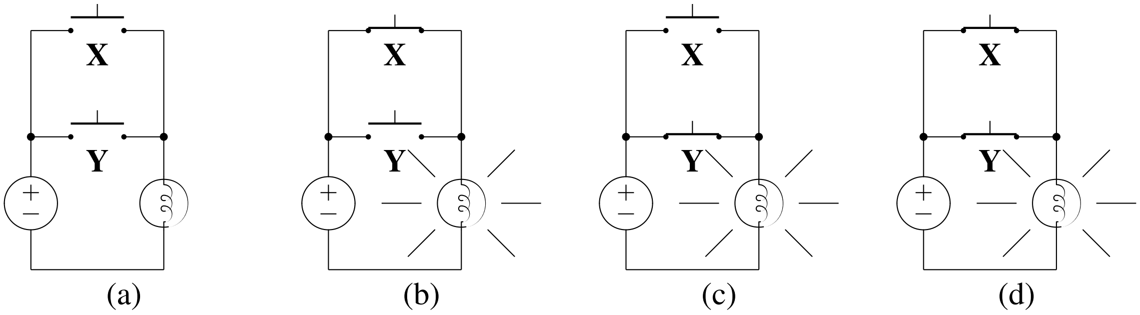


Figure 1.4: Alternate two switch flashlight schematic. (a) Switches **X** and **Y** off. (b) Switches **X** on and **Y** off. (c) Switches **X** off and **Y** on. (d) Switches **X** and **Y** on.

1.4 Boolean exclusive-OR function

The AND configuration allows us to turn the light bulb on with switch **X**, but only if switch **Y** is also on. Conversely, the OR configuration allows us to turn the light bulb off with switch **X**, but only if switch **Y** is off. What if we want to turn the light bulb on and off with either switch without regards to the other?

The solution in this case is to use a *toggle* switch instead of the *on-off* switches we have used up to now. The toggle switch is shown in Figure 1.5 in its two possible configurations.

Figure 1.5: Toggle switch. (a) Switch up. (b) Switch down.

Figure 1.6 shows the new configuration using the two toggle switches. This circuit is similar to Figure 1.3, except that when the switches are in their up position (which corresponds to the off position in Figure 1.3), there is an alternate parallel path from the voltage source to the light bulb. Consequently, the light bulb is on if the switches are either both up or both down. The function we have implemented is the boolean exclusive-OR.

We have achieved our objective. If the light bulb is on (as in Figures 1.6(a) and (d)), it can be turned off by changing the position of either switch. If the light bulb is off (as in Figures 1.6(b) and (c)), it can also be turned on by changing the position of either switch.

1.5 Three switch configurations

It is straightforward to implement a three input AND by placing three on-off switches in series, or a three input OR by placing three on-off switches in parallel, but making a three input exclusive-OR requires the introduction of yet another switch. This is the crossbar switch, and is shown in its two possible configurations in Figure 1.7. In the parallel configuration, the switch connects terminal a to terminal 1 and terminal b to terminal 2. In the crossed configuration, the switch connects terminal a to terminal 2 and terminal b to terminal 1.

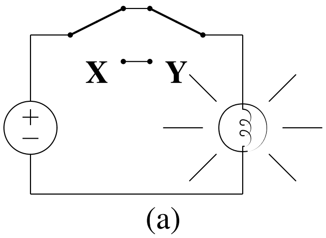


Figure 1.6: Two toggle switches flashlight schematic. (a) Switches **X** and **Y** up. (b) Switches **X** down and **Y** up. (c) Switches **X** up and **Y** down. (d) Switches **X** and **Y** down.

Figure 1.7: Crossbar switch. (a) Switch parallel. (b) Switch crossed.

To make the three input exclusive-OR, we use two toggle switches and one crossbar switch as shown in Figure 1.8. Since we now have three switches, and each switch can be in one of two states, there are a total of eight (2^3) states for the circuit.

If we were to extend the exclusive-OR to n inputs, we would have a total of 2^n states for the circuit, and we would need two toggle switches and $n - 2$ crossbar switches. However, toggle switches are just crossbar switches with one terminal unconnected, just as on-off switches are just toggle switches with one terminal unconnected. Hence we can also say that an n input AND, OR or exclusive-OR requires n (crossbar) switches.

In practice, crossbar switches are rather rare. Fortunately, a crossbar switch can be made out of a common *double-pole double-throw* (DPDT) switch as shown in Figure 1.9(d). A DPDT switch is composed of two mechanically linked *single-pole double-throw* (SPDT or toggle) switches, as shown in Figure 1.9(c). Following this nomenclature, a on-off switch is a *single-pole single-throw* (SPST) switch (Figure 1.9(a)).

In general, any boolean function can be implemented with a suitable arrangement of switches, but more complex functions may require larger mechanically linked switches (3PDT, 4PDT, *etc*).

1.6 Relays

One problem with designing boolean functions strictly with mechanical switches is that we cannot connect the output of one function to the input of another, because the light bulb by itself is not

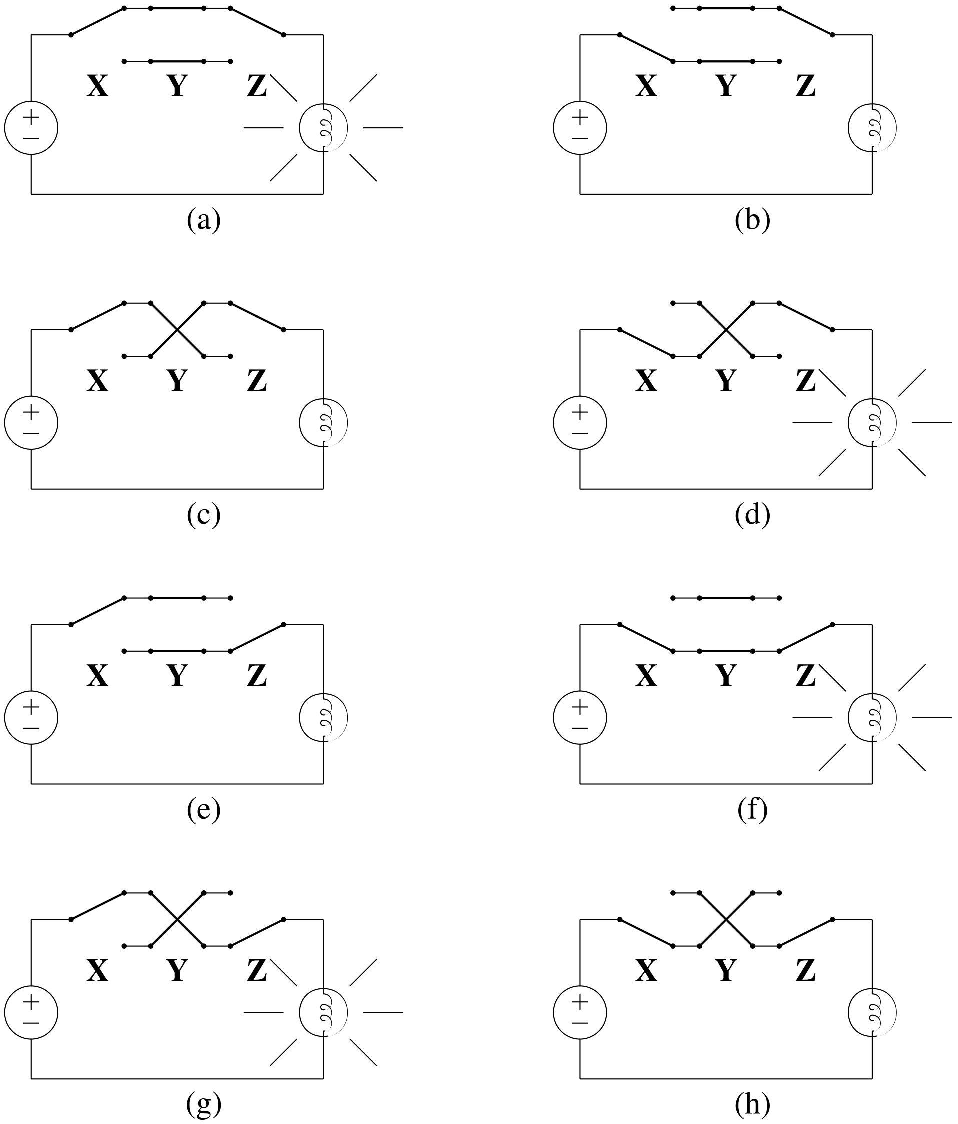


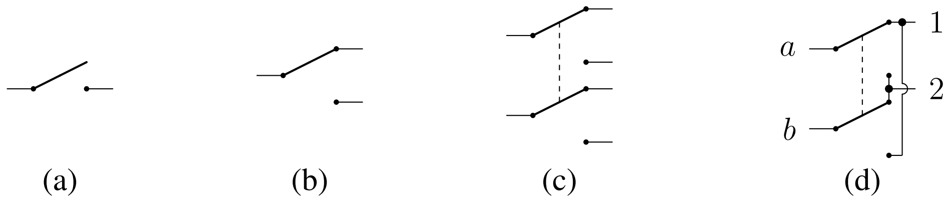
Figure 1.8: Three switch flashlight schematic.

capable of actuating a mechanical switch. What is needed is an electrically controlled switch.

One common example of an electrically controlled switch is the electromagnetic relay. This device uses an electromagnet (which can be as simple as a wire wound around a soft iron bar) to attract the contact plate of a mechanical switch. When the contact plate is drawn to the electromagnet, the switch may be opened or closed, depending on the mechanical arrangement of the device. When the electromagnet is deactivated, a spring returns the contact plate to its original position.

The electromagnet can also be used to control SPDT, DPDT, and crossbar switches, as well as more complex switches, provided that these switches have only two positions.

By replacing the light bulb in the previous examples with electromagnets controlling other switches, it is possible to design complex Boolean circuits. However, these circuits are beyond the scope of this text.



(a) (b) (c) (d)

Figure 1.9: Common switches. (a) SPST (on-off) switch. (b) SPDT (toggle) switch. (c) DPDT switch. (d) DPDT crossbar switch.

Chapter 2

MOS transistors as switches

2.1 Capacitive loads

In operation, a light bulb is similar to a resistor: when a voltage is applied across its two terminals, a current flows through it (and incidentally cause the filament to give off light). For these devices, the amount of current that flows through the device is only a function of the applied voltage.

However, there are a large number of devices for which the current is a time dependent function of the applied voltage. Of these, the reactive components are the simplest. Reactive components can be either inductive or capacitive in nature.

Inductive components, such as magnetic coils, exhibit a current which is related to the time integral of the applied voltage. Capacitive components exhibit a current which is related to the time derivative of the applied voltage.

One of the direct effects of a capacitor's reactive nature is its charge retention. This is shown in Figure 2.1, where the light bulb of Figure 1.2 has been replaced with a capacitor.

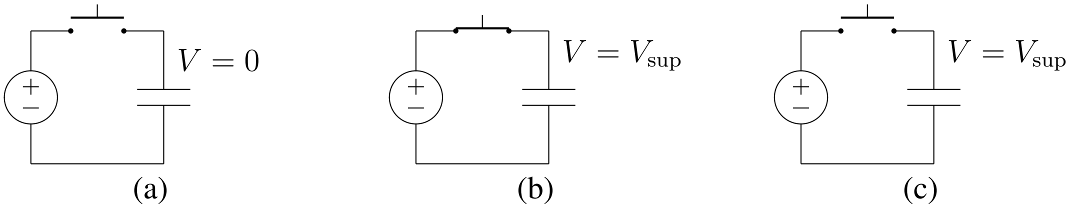
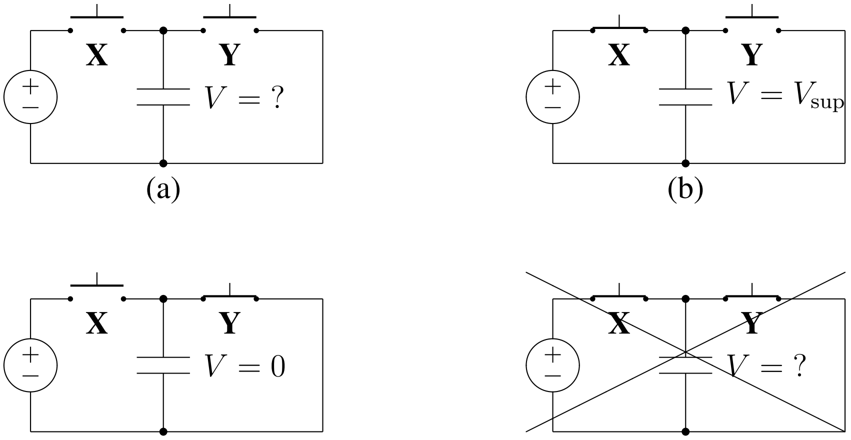


Figure 2.1: Time sequence for a capacitive load. (a) Switch off. (b) Switch on. (c) Switch off again.

The time sequence is as follows. Assume that the capacitor initially has no voltage across it ($V = 0$, Figure 2.1(a)). When the switch is closed, the capacitor rapidly charges up to the same voltage V_{sup} as the voltage supply (Figure 2.1(b)). Thus far, the behavior is similar to the light bulb. However, when the switch is opened again (Figure 2.1(c)), the charge on the capacitor remains, and therefore the voltage on the capacitor continues to be V_{sup}.

How can we restore the circuit to the state it was in in Figure 2.1(a)? The solution is to add a second switch to set the voltage of the capacitor to the voltage it had in Figure 2.1(a). Since this voltage was zero, we do not need an explicit second voltage supply, so we can introduce the second switch as shown in Figure 2.2(a).



(a) (b)

(c) (d)

Figure 2.2: Two switches with capacitive load. (a) Switches **X** and **Y** off. (b) Switches **X** on and **Y** off. (c) Switches **X** off and **Y** on. (d) Switches **X** and **Y** on, producing a short circuit.

In this new configuration, if only switch **X** is on, the capacitor is charged to the supply voltage V_{sup} (Figure 2.2(b)), and if only switch **Y** is on, the capacitor is discharged to zero (Figure 2.2(c)). If both switches are off, the capacitor voltage remains at the voltage it was last charged to (Figure 2.2(a) again).

Although we have solved one problem with regards to the capacitor's charge, we have introduced a new potential problem. In this new switch configuration, it is also possible for both switches to be on as in Figure 2.2(d). This is a highly undesirable configuration, because there is a direct path from one end of the voltage source to the other. Since there are no current limiting devices in the path, a large amount of current can flow, potentially damaging the circuit's wires or components. In this configuration, the power supply is said to be *short circuited*.

2.2 MOSFET switches

Metal-Oxide-Semiconductor Field-Effect Transistors (MOSFET) can be used as switches, and have the property that their control terminal behaves like a capacitor. The control terminal is called the *Gate*, and the potential at the Gate terminal determines whether or not current can flow between the MOSFET's *Source* and *Drain* terminals.

In modern silicon fabrication processes, MOSFETs come in two complementary types: the N-channel MOSFET (or NFET for short) and the P-channel MOSFET (or PFET for short). The NFET turns on with a positive voltage at the gate, and the PFET turns on with a negative voltage at the gate.

MOSFETs also have a fourth terminal called the *Bulk* or the *Body*. The bulk terminal is almost always connected to either the positive or negative terminal of the circuit's power supply. Some modern state-of-the-art processes which are fabricated in Silicon-on-Insulator (SOI) even leave the bulk terminal unconnected. For these reasons, connections to the bulk terminal are not usually explicitly drawn in schematics, unless the connection is unusual in some manner.

Figures 2.3 and 2.4 shows some of the common symbols used for MOSFETs in schematics.

In virtually all cases, the source and drain terminals are indistinguishable from one another, and hence they are defined by the direction of the current flow I in the MOSFET.

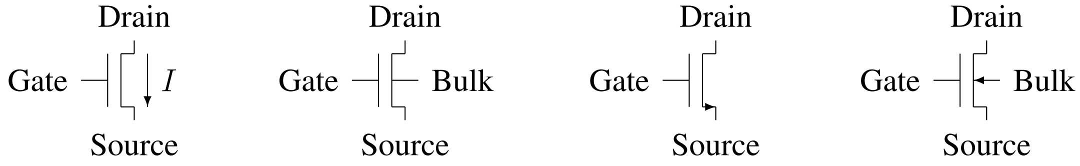


Figure 2.3: Common symbols for N-channel MOSFETs (NFETs).

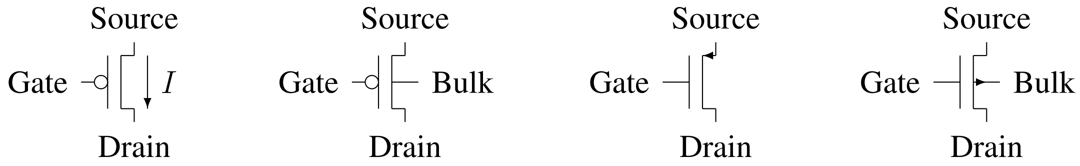


Figure 2.4: Common symbols for P-channel MOSFETs (PFETs).

2.3 Labeling nodes and power supplies in schematics

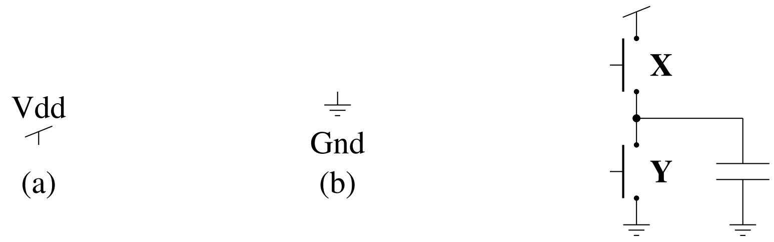
A typical electronic circuit may contain many elements and can result in a very large schematics with many drawn wires. Since the complete schematic may not fit on a single piece of paper, different parts of the schematic will need to be drawn on separate pages.

To indicate that a point in one part of the schematic is connected to another point in another part of the schematic, the node is given a label. The same label can then be used in multiple locations on the schematic to indicate all the circuit points which are connected to the same node.

Although it is usually easier to follow wire on a schematic, if there are too many of them, the resulting rat's nest can render the schematic illegible. Hence,even if the entire schematic fits on a single page, labeling nodes is still used to reduce the number of drawn wires thereby simplifying the appearance of the schematic.

Circuit power supplies are a special case of node labeling. Since there is usually very few power supplies in a circuit, and often only one, and since a large number of components are connected to each power supply, special symbols are used instead of labels or in addition to them to identify circuit points connected to the positive and negative terminals.

The positive terminal of the main power supply in MOSFET circuits is usually called Vdd. This is because older MOSFET circuits were composed entirely of NFETs (PFETs appeared later) and hence the positive terminal of the main power supply was connected exclusively to the drain terminal of NFETs. Similarly, the negative terminal of the main power supply is usually called Vss or Gnd (for ground). The traditional symbols for Vdd and Gnd are shown in Figure 2.5(a) and (b), and the circuit in Figure 2.2(a) is shown redrawn using these symbols in Figure 2.5(c). Note how the switches have been rotated so that the Vdd symbol can be located at the top of the schematic, and the Gnd symbol at the bottom.



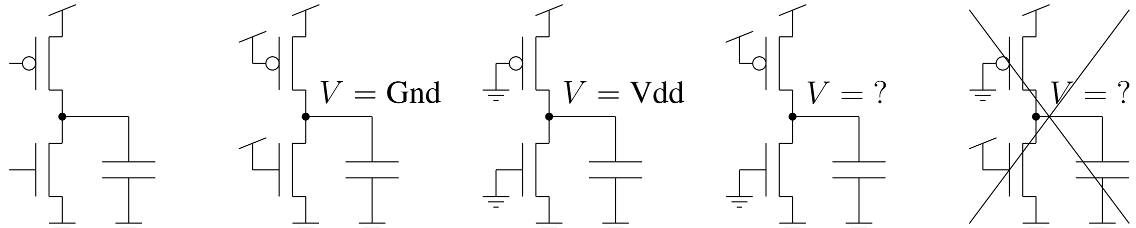
(a) (b) (c)

Figure 2.5: Power supply symbols and their use. (a) Positive terminal. (b) Negative terminal. (c) With switches.

2.4 Pull-up and pull-down MOSFET configuration

A further distinction between NFETs and PFETs is that NFETs are good at discharging capacitive loads to Gnd but not too good at charging capacitive loads to Vdd, whereas PFETs are good at charging capacitive loads to Vdd but not too good at discharging capacitive loads to Gnd.

Hence, switch **X** in Figure 2.5(c) is best replaced by a PFET and switch **Y** is best replaced by an NFET. The resulting circuit is shown in Figure 2.6(a).



(a) (b) (c) (d) (e)

Figure 2.6: Two MOSFET pull-up and pull-down configuration.

If the gates of both MOSFETs are connected to Vdd as in Figure 2.6(b), then the NFET is turned on and the PFET is turned off. The result is that the capacitor is discharged or pulled-down to Gnd. If the gates of both MOSFETs are connected to Gnd as in Figure 2.6(c), then the NFET is turned off and the PFET is turned on. The result is that the capacitor is charged or pulled-up to Vdd.

If the gate of the NFET is connected to Gnd and the gate of the PFET is connected to Vdd as in Figure 2.6(d), then both MOSFETs are turned off and the capacitor voltage remains at the voltage it was last charged to. However, this state is undesirable because the MOSFETs are not perfect switches. When a MOSFET is turned off, a tiny amount of leakage persists from source to drain. Because this leakage is highly variable from device to device, we cannot predict whether the capacitor voltage will remain the same or whether it will tend to drift over time to either Vdd or Gnd.

If the gate of the NFET is connected to Vdd and the gate of the PFET is connected to Gnd as in Figure 2.6(e), then both MOSFETs are turned on and we have a short circuit situation as in Figure 2.2(d). Fortunately, since the MOSFETs are not perfect switches, they will limit the short circuit current, typically to a magnitude that will be insufficiently large to damage the MOSFET themselves. However, if there are many such short circuits in an integrated circuit, they can prevent other portions of the circuit from operating properly, and they may even cause wires to fuse open, resulting in permanent damage to the integrated circuit.

2.5 Other two MOSFET configurations

Given the two switch circuit of Figure 2.2, there are three more ways that we can replace switches X and Y with MOSFETs. However, all three of these configurations involve using an NFET to pull-up or a PFET to pull-down (or both).

If an NFET is used as a pull-up as shown in Figure 2.7(a), it will take a very long time for the capacitor voltage to reach Vdd. In practice, relative to the speeds at which digital circuits normally operate, the capacitor voltage will never reach Vdd. It is difficult to predict whether the voltage that is reached at the capacitor will be enough to turn on an other NFET. Furthermore, if the other NFET is also being used as a pull-up, the voltage reach by the other NFET will be even lower still.

(a) (b)

Figure 2.7: Undesirable MOSFET configurations. (a) NFET as pull-up. (b) PFET as pull-down.

Similarly, if a PFET is used as a pull-down as shown in Figure 2.7(b), it will take a very long time for the capacitor voltage to reach Gnd. Again, in practice, relative to the speeds at which digital circuits normally operate, the capacitor voltage will never reach Gnd, and it is difficult to predict whether the voltage that is reached at the capacitor will be enough to turn on an other PFET. Furthermore, if the other PFET is also being used as a pull-down, the voltage reach by the other PFET will be even higher still.

2.6 Boolean conventions

By using combinations of multiple MOSFETs we can create a small subcircuit with one or more inputs (connected to the gates of the MOSFETs) and one or more outputs, for which various patterns of inputs (in which each input is either pulled-up to Vdd or pulled-down to Gnd) causes the outputs themselves to be pulled-up or pulled-down. Hence we have the building blocks for making binary logic gates whose inputs and outputs are compatible, and we can therefore take advantage of all the theorems related to Boolean logic.

By convention we will consider a voltage of Vdd to represent a Boolean logic or binary 1 and a voltage of Gnd to represent a Boolean logic or binary 0. With this convention, we can create a table of the equivalence between MOSFETs and switches as shown in Figure 2.8.

Figure 2.8: Boolean equivalence between MOSFETs and switches.