

The Johns Hopkins University
Department of Electrical and Computer Engineering

520.621 — Nonlinear Systems — Spring 2005

Problem Set #1

Due: Wednesday, February 9, 2005, at the end of class. Late solutions are penalized 20% per day or part thereof.

Instructions: The solutions you hand in should be well-organized, precisely stated and clearly written. Start each solution on a new page, and staple the sheets in the correct order. If I can not read your solution, I will not try to grade it.

Ethics statement: These problems are meant to be done individually; any discussion with other students should be limited to clarification of the questions. Consulting solution manuals, or solutions from previous years, will be considered a violation of academic ethics and will be punished.

Problems:

1. Consider the differential equation

$$\dot{x}(t) = -\operatorname{sgn}[x(t)], \quad x(0) = x_0$$

where

$$\operatorname{sgn}[x] = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

[note that this is a slightly different definition than that used in class]. If a solution is defined to be continuously-differentiable, investigate the existence of a solution.

2. Khalil, problem 1.1.
3. Khalil, problem 1.5.
4. Consider the nonlinear system described by

$$\dot{x} = x(1 - x) - h \frac{x}{a + x}$$

Show that the system can have one, two or three equilibria, depending on the values of a and h . Classify the stability of the fixed points in each case.