The Johns Hopkins University
Department of Electrical and Computer Engineering

520.353 — Control Systems — Fall 1995

Final Exam

Name: ______________________________________

SSN (last 4 digits): ____________

Instructions (Read carefully!)

1. This exam is closed-book, closed-notes, but the use of three 8.5×11 inch, handwritten sheets is allowed. **No calculators are allowed.**
2. Where proofs are required, give neat, step-by-step solutions that include all details and are based on first principles. Where computations are required, give neat, step-by-step solutions that include all the details. Where graphs are needed, make sure that they are neat, and clearly labelled.
3. Attept all questions.
4. Good Luck!

Marks

Question	Maximum	Marks
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Useful formulae

$$\mathcal{L}[e^{-bt}\sin(at)] = \frac{a}{(s+b)^2+a^2}, \quad \mathcal{L}[e^{-bt}\cos(at)] = \frac{s+b}{(s+b)^2+a^2}$$

Problems

1. A student found herself in a lab with an inverted pendulum whose linearized model had a transfer function given by:
$$P(s) = \frac{2}{s^2 - 9}$$
In order to stabilize this system, the student built a unity-gain feedback controller with transfer function:
$$C(s) = \frac{s-3}{s+2}$$
When she let go of the pendulum, it quickly fell. Why? Give at least three possible explanations of what might have gone wrong. Also, help the student out by designing a better controller. **NB**. For 20% of your final exam mark, I'm expecting a detailed and well thought-out explanation. Make sure that you present your thoughts clearly — a portion of your grade for this question will depend on how well you do this.

2. Consider the plant $P(s) = \dfrac{10}{(s+1)(s+10)}$.
   (a) Find a controller $C(s)$ such that:
      i. The steady state error due to a unit step is ≤ 0.1; *and*
      ii. The phase margin $PM \geq 90°$.
   (b) Find the gain margin of the system with the controller you used in part (a).

3. Consider a system with
$$kPCF(s) = k\frac{s-1}{s^2 - s + 2}$$
   (a) Plot the Nyquist diagram for PCF.
   (b) Use the diagram from part (a) to find the range of gains k which will stabilize the system.

4. Consider a system with state-space representation given by:
$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1/2 & 1 \end{bmatrix}$$
Find e^{At} and use this to find $y(t)$ for $t \geq 0$ given $r(t) = 0$ and $x'(0) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$.

5. Consider the differential equation
$$\ddot{y}(t) - p\dot{y}(t) = \dot{r}(t) - zr(t)$$
with zero initial conditions. (NB. Here, p and z are just constants.)
   (a) Using
$$x_1(t) := y(t), \qquad x_2(t) := \dot{y}(t) - r(t)$$
   write the differential equation in standard state-space form.
   (b) Investigate the controllability and observability properties of the system you obtained in (a) as a function of the parameters z and p.
   (c) Find the transfer function $G(s) := Y(s)/R(s)$ and relate the controllability/observability properties you found in (b) to $G(s)$.