The Johns Hopkins University
Department of Electrical and Computer Engineering

520.353 — Control Systems — Fall 2007

Term Test No. 2.

Name: ____

Instructions (Read carefully!)

1. This exam is closed-book, closed-notes, but the use of one 8.5×11 inch, handwritten sheet and one photocopied Laplace Transform table is allowed. **No calculators are allowed.**
2. Where proofs are required, give neat, step-by-step solutions that include all details and are based on first principles. Where computations are required, give neat, step-by-step solutions that include all the details. Where graphs are needed, make sure that they are neat, and clearly labelled.
3. Good Luck!

Marks

Question	Maximum	Marks
1	10	
2	10	
3	10	
Total	30	

Problems

1. Consider the standard feedback system with:

$$P(s) = \frac{s^2+3}{s^2-1}, \qquad C(s) = \frac{k}{s}, \qquad F(s) = 1.$$

(a) For the root locus for this system, find the arrival angles at the zeros.

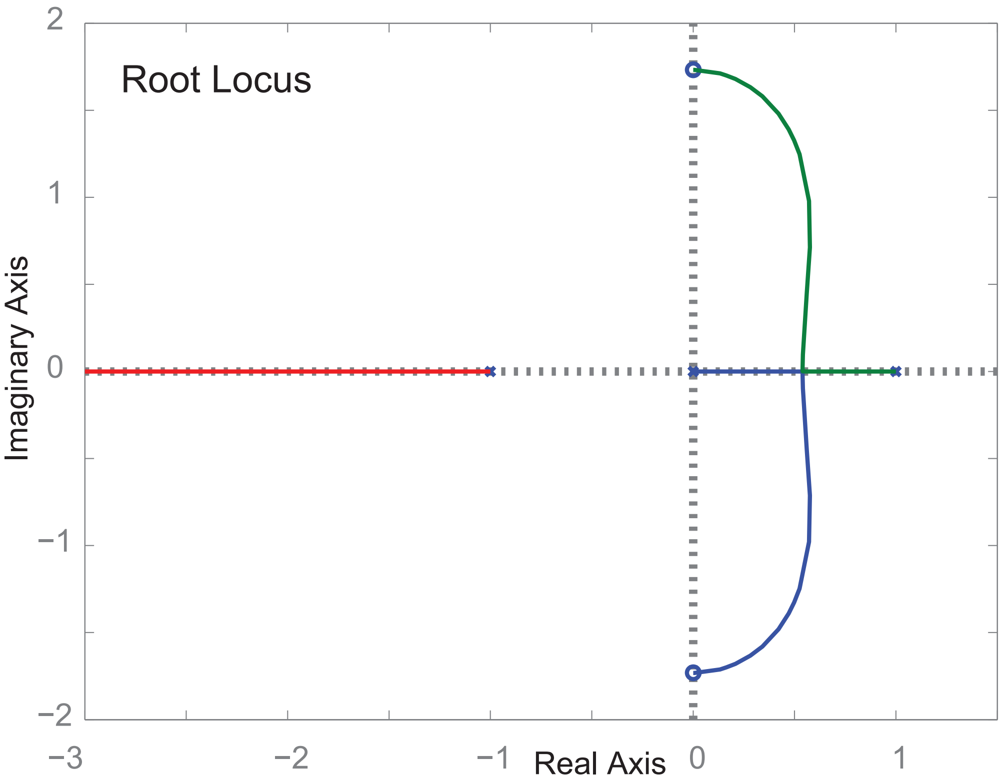
Solution. Let θ be the arrival angle at the zero at $+\sqrt{3}j$. Then:

$$\theta + 90^\circ - 60^\circ - 120^\circ - 90^\circ = 180^\circ \Rightarrow \theta = 360^\circ = 0^\circ.$$

By symmetry, it is also 0° at the zero at $-\sqrt{3}j$.

(b) Plot the root locus for this feedback system. (Neatness, and proper labels are important!)

Solution.



2. Consider the standard feedback system with:

$$k(PCF)(s) = k\frac{(1+s)(4+s^2)}{s(1-s)(1+s^2)}$$

Sketch the Nyquist plot of PCF and use the Nyquist stability criterion to the range of gains k for feedback stability.

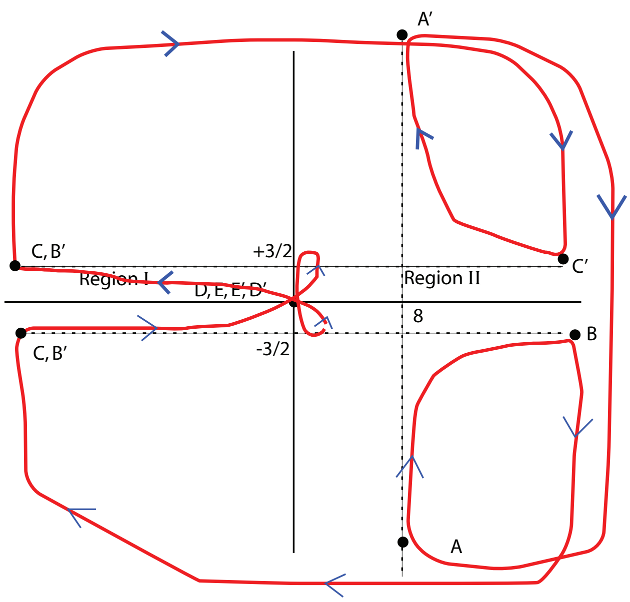
Solution. We first need to write this as

$$\begin{aligned}(PCF)(j\omega) &= \frac{4-\omega^2}{1-\omega^2} \times \frac{1+j\omega}{j\omega(1-j\omega)} \\ &= \frac{4-\omega^2}{1-\omega^2} \times \frac{1+j\omega}{j\omega(1-j\omega)} \times \frac{1+j\omega}{1+j\omega} \\ &= \frac{4-\omega^2}{\omega(1-\omega^2)} \times \frac{1-\omega^2+2j\omega}{j(\omega^2+1)} \\ &= \frac{4-\omega^2}{\omega(1-\omega^2)} \times \frac{2\omega+j[\omega^2-1]}{\omega^2+1} \\ &= \frac{2(4-\omega^2)}{1-\omega^4} + j\frac{(\omega^2-4)}{\omega(1+\omega^2)}\end{aligned}$$

The Nyquist contour in this case requires three indentations, at 0 and at $\pm j$.

Form a table:

Label	ω	Real	Imag	Slope=$(\omega^2-1)/[2\omega]$
A	0^+	8	$-\infty$	
B	1^-	$+\infty$	$-3/2$	
C	1^+	$-\infty$	$-3/2$	
D	2	0	0	$3/4$
E	$+\infty$	0^+	0^+	$+\infty$



You need 1 CCW encirclement, but both regions I&II have 0 or 1 CW encirclements. Thus, no k makes this closed-loop system stable.

3. Consider the standard feedback system with:

$$(PCF)(s) = \frac{1000(s+1)}{s(s+10)^2}$$

(a) In the graphs provided, sketch the (piece-wise linear approximation) Bode plots for PCF.

Solution. Rewrite as: $\dfrac{1000(s+1)}{s(s+10)^2} = \dfrac{10(s+1)}{s(s/10+1)^2} = \dfrac{1 \times 2}{3 \times 4}$.