The Johns Hopkins University
Department of Electrical and Computer Engineering
520.353 — Control Systems
Fall 2007

Problem Set Number 8

Due: Wednesday, December 5, 2007 in class.

Problems:

1. Determine the stability properties (i.e. are the matrices stable and/or asymptotically stable) of:

(a) $A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$, (b) $A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$,

(c) $A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, (d) $A_4 = \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$,

(e) $A_5 = \begin{bmatrix} 1 & 2 \\ -2 & -2 \end{bmatrix}$, (f) $A_6 = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$.

2. The linearized equations of motion for a satellite are

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The inputs u_1 and u_2 are the radial and tangential thrusts, the state variables x_1 and x_3 are the radial and angular deviations from the reference (circular) orbit, and the outputs y_1 and y_3 are the radial and angular measurements, respectively.

(a) Show that the systems is controllable using only a single input. Which one is it?

(b) Show that the system is observable using only one measurement. Which one?

3. Consider the differential equation

$$\ddot{y}(t) - p\dot{y}(t) = \dot{r}(t) - zr(t)$$

with zero initial conditions. (NB. Here, p and z are just constants.)

(a) Using

$$x_1(t) := y(t), \qquad x_2(t) := \dot{y}(t) - r(t)$$

write the differential equation in standard state-space form.

(b) Investigate the controllability and observability properties of the system you obtained in (a) as a function of the parameters z and p.

(c) Find the transfer function $G(s) := Y(s)/R(s)$ and relate the controllability/observability properties you found in (b) to $G(s)$. Specifically, when do yo lose observability and/or controllability?

4. Give an example of a fourth order system (A, B, C) that satisfies all of these conditions:

   (a) The system is controllable;

   (b) the system is not observable;

   (c) the matrix A is stable;

   (d) the matrix A is not asymptotically stable;

   (e) the transfer function

   $$G(s) = C(sI - A)^{-1}B$$

   is UBIBOS and of second order.

   Hint: Consider systems already in modal form (i.e. with a diagonal A matrix).