The Johns Hopkins University
Department of Electrical and Computer Engineering

520.454 — Control Systems Design — Spring 2007

**Problem Set #0**

**Problems**

1. Suppose that the input to a linear, time-invariant system was

$$u(t) = e^t \sin(2t), \qquad t \geq 0$$

and this resulted in the output

$$y(t) = e^{\alpha t} \cos(2t) + e^t \sin(2t) + e^{-3t}, \qquad t \geq 0.$$

   (a) Let $\alpha = 1$. Find the transfer function of the system. Is the system bounded-input-bounded-output stable?

   (b) Let $\alpha = 0$. Find the transfer function of the system. Is the system bounded-input-bounded-output stable in this case?

2. Let

$$k(PCF)(s) = k\frac{(s+1)^2 + 1}{s(s^2 - 1)}$$

   (a) Draw the Nyquist diagram for $(PCF)(s)$.

   (b) Use the Nyquist stability criterion to determine for which range of $k$ the feedback system is internally stable.

3. Consider the standard closed-loop system with

$$P(s) = \frac{1}{s+1}, \qquad C(s) = K_1 + \frac{K_2}{s}, \qquad F(s) = 1.$$

   Find gains $K_1$ and $K_2$ so that the closed-loop poles lie in the left half-plane $\operatorname{Re} s < -1$ and the steady-state error due to a unit ramp is less than 10%.

4. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu, \qquad x(0) = x_0 \\ y &= Cx \end{aligned}$$

   where

$$A = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix},$$

   (a) Find $e^{At}$.

   (b) Is the system stable? Asymptotically stable? Bounded-input-bounded-output stable?

   (c) Suppose that $u \equiv 0$. Find $y(t)$ for $t \geq 0$.

5. For the system,

$$\begin{aligned} \dot{x} &= Ax + Bu, \qquad x(0) = x_0 \\ y &= Cx \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \alpha \end{bmatrix}, \quad C = \begin{bmatrix} \beta & 1 & 0 & 0 \end{bmatrix}$$

(a) Determine what values of $\alpha$ make the system controllable.

(b) Determine what values of $\beta$ make the system observable.

(c) When is the realization minimal?