

# Self-Test

520.460, Digital Communications I

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1. Let  $x$  be a continuous random variable with probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-m)^2/2\sigma^2} \quad (1)$$

Find the mean and variance of  $x$ .

2. Let  $x$  be as in the previous problem and  $y = x^2$ . Determine the probability  $P[y < 2]$ .
3. Let  $n$  be a random variable with density function as in (1) with mean = 0.
  - (a) What is the probability  $P[n > T]$ ?
  - (b) What is the probability that  $n < -T$ ?
  - (c) Write the probability that  $n > T$  in terms of the answer to the previous question.
4. Consider a biased coin that produces "heads" with probability  $p$  and "tails" with probability  $(1 - p)$ . If the coin is tossed  $N$  times, what is the probability that "heads" comes up  $t$  times?
5. Suppose you observe the signal  $\cos(2\pi ft + \theta)$  where  $\theta$  is a random phase angle that is uniformly distributed on the interval  $[0, 2\pi)$ . What is the probability  $P[\pi/2 < \theta < \pi]$ ?
6. Consider an infinitely long sequence of signals  $a_1, a_2, a_3, \dots$  each of which has amplitude -3 with probability 3/8, -1 with probability 1/8, +1 with probability 1/8, and +3 with probability 3/8. What is the average power in this sequence?
7. Let  $x$  and  $y$  be independent, discrete random variables with probability mass functions  $p(x)$  and  $q(y)$ , respectively. If  $z = y + x$ , what is the probability mass function for  $z$ ?