Notes for Signals and Systems

9.3 Operations on Signals

Discrete-time, periodic signals are completely determined by the fundamental frequency, ω_o, or fundamental period, N_o, and any N_o consecutive Fourier coefficients, say, $X_0, X_1, ..., X_{N_o-1}$. Thus the signal is described in terms of its frequency content. This raises the possibility of interpreting various time-domain operations on signals as operations on the frequency-domain description. However, rather than give a lengthy treatment of this issue, we will simply discuss a few examples.

Example 1 Given a signal

$$x[n] = \sum_{k=<N_o>} X_k \, e^{jk\omega_o n}$$

suppose a new signal is obtained by the index shift

$$\hat{x}[n] = x[n - n_o]$$

where n_o is a fixed integer. Clearly a shift does not change periodicity, or the fundamental period, or fundamental frequency. Therefore we can compute the Fourier coefficients for $\hat{x}[n]$ from the standard formula:

$$\hat{X}_k = \frac{1}{N_o} \sum_{n=<N_o>} \hat{x}[n] e^{-jk\omega_o n} = \frac{1}{N_o} \sum_{n=<N_o>} x[n - n_o] e^{-jk\omega_o n}$$

Changing the summation index from n to $m = n - n_o$,

$$\hat{X}_k = \frac{1}{N_o} \sum_{m=<N_o>} x[m] e^{-jk\omega_o (m+n_o)} = e^{-jk\omega_o n_o} \frac{1}{N_o} \sum_{n=<N_o>} x[n] e^{-jk\omega_o m}$$

$$= e^{-jk\omega_o n_o} X_k$$

Notice that the magnitude spectrum of the signal is unchanged by time shift, since, regardless of the integer value of *k*,

$$\left|\hat{X}_k\right| = \left|e^{-jk\omega_o N_o}\right| \left|X_k\right| = \left|X_k\right|$$

In simple cases, such as time-index shift, it is possible to ascertain the effect of the operation on the Fourier coefficients by inspection of the representation. Indeed, with $x[n]$ as given above, it is clear that

$$\hat{x}[n] = x[n - n_o] = \sum_{k=<N_o>} X_k \, e^{jk\omega_o (n-n_o)} = \sum_{k=<N_o>} e^{-jk\omega_o n_o} X_k \, e^{jk\omega_o n}$$

and we simply recognize the form of the expression and the corresponding Fourier coefficients $\hat{X}_k$.

Example 2 Suppose $\hat{x}[n] = x[-n]$. Again, the fundamental frequency does not change, and the Fourier coefficients for $\hat{x}[n]$ are given by

$$\hat{X}_k = \frac{1}{N_o} \sum_{n=<N_o>} \hat{x}[n] e^{-jk\omega_o n} = \frac{1}{N_o} \sum_{n=<N_o>} x[-n] e^{-jk\omega_o n}$$

Changing the summation index to $m = -n$ gives

$$\widehat{X}_k = \frac{1}{N_o} \sum_{m=<N_o>} x[m] e^{-jk\omega_o(-m)} = \frac{1}{N_o} \sum_{m=<N_o>} x[m] e^{-j(-k)\omega_o n}$$

$$= X_{-k}$$

This conclusion also could be reached by inspection of the representation.

Further discussion of operations on discrete-time signals can be found in the demonstration

DTFS Properties