Notes for Signals and Systems

9.2 Spectra of DT Signals

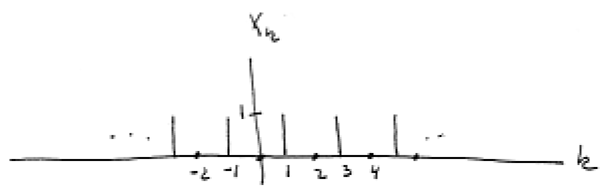
For a real, periodic, DT signal $x[n]$, the frequency content of the signal is revealed by the coefficients in the DTFS expression

$$x[n] = \sum_{m=<N_o>} X_m \, e^{jm\omega_o n}$$

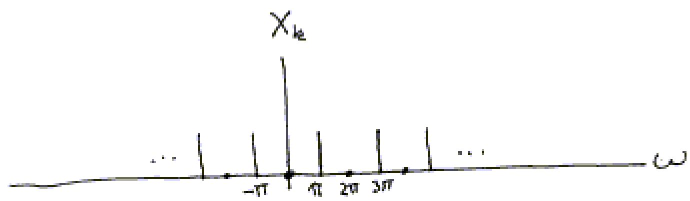
The following graphical displays of these coefficients define various *spectra* of $x[n]$.

The *magnitude spectrum* of $x[n]$ is a line plot of $|X_m|$ vs the index *m*, or vs $m\omega_o$ on a frequency axis. The *phase spectrum* of $x[n]$ is a similar plot of $\angle X_m$, usually on an angular range from $-\pi$ to π. Finally, when the DTFS coefficients are all real, the *amplitude spectrum* of $x[n]$ is simply a plot of the coefficients X_m.

Example In Section 9.1 we computed the DTFS coefficients of $x[n] = (-1)^n$ as $X_0 = 0,\ X_1 = 1,$ and $X_{k+2} = X_k$, for other values of *k*. In this case the amplitude spectrum of the signal is simply



or, in terms of a frequency axis, since $\omega_o = \pi$,

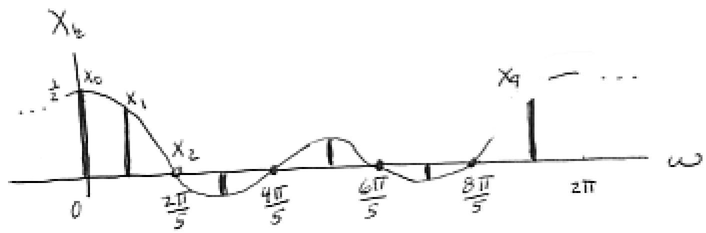


Since π corresponds to the highest frequency in discrete time, we note the obvious fact that $x[n]$ is a high-frequency signal. Finally, for this simple case, the magnitude spectrum is identical to the amplitude spectrum, and the phase spectrum is zero.

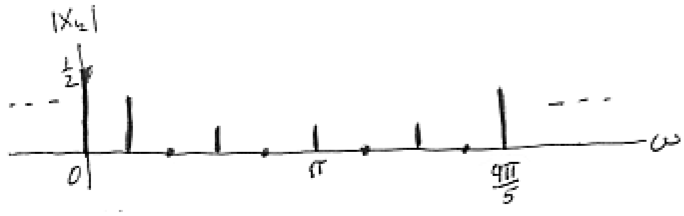
Example The second example in Section 9.1, a periodic train of width $2N_1 + 1$ clumps of unit-height lollypops, is considerably more complicated, though again the DTFS coefficients are real. Choosing $N_1 = 2,\ N_o = 10$, the coefficients are given as an evaluation of an *aliased sinc* envelop by

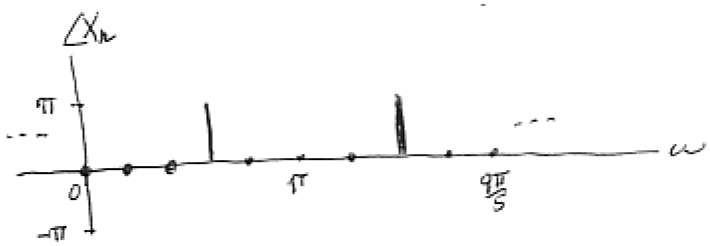
$$X_m = \frac{1}{10} \frac{\sin(5\omega/2)}{\sin(\omega/2)}\bigg|_{\omega = m\pi/5}$$

The envelope function is zero when $5\omega / 2 = k\pi$, for nonzero integer *k*, that is, for $\omega = 2k\pi / 5$. Sketching this envelope function yields the amplitude spectrum shown below.



Correspondingly, the magnitude and phase spectra are shown below.





While there is some high-frequency content in $x[n]$, in particular the component at frequency π, there is more low-frequency content as indicated by the components near the frequencies zero and 2π. Finally, it should be noted that these spectra repeat outside the frequency ranges shown.

To explore the notion of spectra in more detail, consult the demonstration *Discrete-Time Fourier Series* linked at the end of Section 9.1.