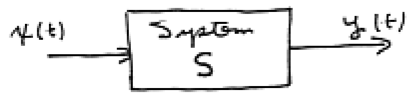
Notes for Signals and Systems

4.1 Introduction to Systems

A continuous-time *system* produces a new continuous-time signal (the *output* signal) from a provided continuous-time signal (the *input* signal). Attempting a formal definition of a system is a tedious exercise in avoiding circularity, so we will abandon precision and rely on the intuition that develops from examples. We represent a system "S" with input signal $x(t)$ and output signal $y(t)$ by a box labeled as shown below.



Since a system maps signals into signals, the output signal at any time t can depend on the input signal values at all times. We use the mathematical notation

$$y(t) = S(x)(t)$$

to emphasize this fact.

Remark There are many notations for a system in the literature. For example, some try to emphasize that a system maps signals into signals by using square brackets,

$$y(t) = S[x(t)]$$

However this still tempts the interpretation that, for example, $y(0) = S[x(0)]$, i.e., $y(0)$ depends only on $x(0)$. Our notation is designed to emphasize that for the input signal "x" the output signal is "$S(x)$," and the value of this output signal at, say $t = 0$, is $y(0) = S(x)(0)$.

Example The running integral is an example of a system. A physical interpretation is a capacitor with input signal the current $x(t)$ through the capacitor, and output signal the voltage $y(t)$ across the capacitor. Then we have, assuming unit capacitance,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

In this case, the output at any time t_1 depends, on input values for all $t \le t_1$. Specifically, at any time t_1, $y(t_1)$ is the accumulated net area under $x(t)$ for $-\infty < t \le t_1$.

In the discrete-time case, we use an entirely similar notation, writing $y[n] = S(x)[n]$ and representing a system by the diagram shown below.