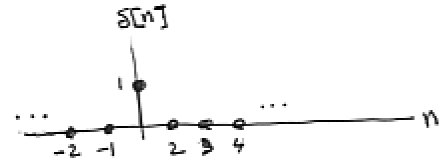
Notes for Signals and Systems

3.2 The Class of DT Singularity Signals

The basic discrete-time singularity signal is the *unit pulse*,

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$$

Contrary to the continuous-time case, there is nothing “generalized” about this simple signal. Graphically, of course, $\delta[n]$ is a lonely lollypop at the origin:



The discrete-time unit-step function is

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

and again there are no technical issues here. In particular, the value of $u[0]$ is unity, unlike the continuous-time case where we decline to assign an immoveable value to $u(0)$. Graphically, we have

The unit step can be viewed as the running sum of the unit pulse,

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Changing summation variable from k to $l = n - k$ gives an alternate expression

$$u[n] = \sum_{l=0}^{\infty} \delta[n-l]$$

This process can be continued to define the *unit ramp* as the running sum of the unit step, though there is a little adjustment involved in the upper limit of the sum, since $u[0] = 1$ and $r[0] = 0$:

$$r[n] = \sum_{k=-\infty}^{n-1} u[k] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

We will have no reason to pursue further iterations of running sums. One reason is that simple discrete-time signals can be written as sums of amplitude scaled and time shifted unit pulses, and there is little need to write signals in terms of steps, ramps, and so on.

Discrete-time singularity signals also have sifting and multiplication properties similar to the continuous-time case, though no “generalized” interpretations are needed. It is straightforward to verify that

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_o] = x[n_o]$$

which is analogous to the continuous-time sift

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_o)\, dt = x(t_o)$$

Also,

$$x[n]\delta[n-n_o] = x[n_o]\delta[n-n_o]$$

is a discrete-time version of the multiplication rule

$$x(t)\delta(t-t_o) = x(t_o)\delta(t-t_o)$$

However, unlike the continuous-time case, the time-scaled unit pulse, $\delta[an]$, where a is a nonzero integer, is identical to $\delta[n]$, as is easily verified.