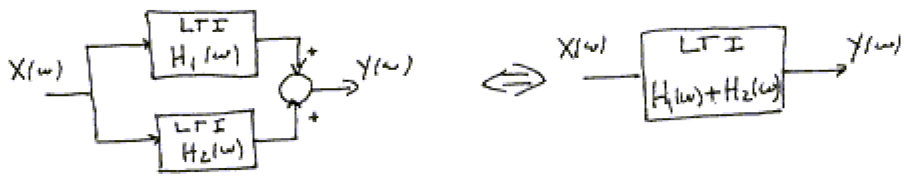
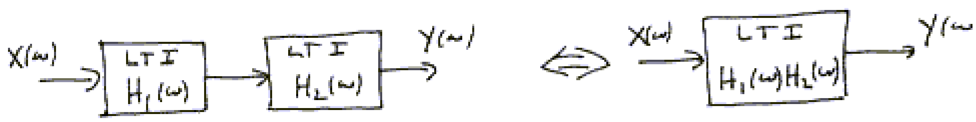
Notes for Signals and Systems

10.8 Fourier Transform and Interconnections of LTI Systems

Interconnections of *stable* LTI systems are conveniently described in terms of frequency response functions, though it must be guaranteed that the overall system also is stable for the overall frequency response function to be meaningful. Assuming this, block diagram equivalences in terms of frequency response functions follow from the time domain results, at least for the first two cases. Namely, for additive parallel connections, where the overall unit-impulse response is the sum of the subsystem unit-impulse responses, and for cascade connections, where the overall unit-impulse response is the convolution of the subsystem unit-impulse responses, we immediately have

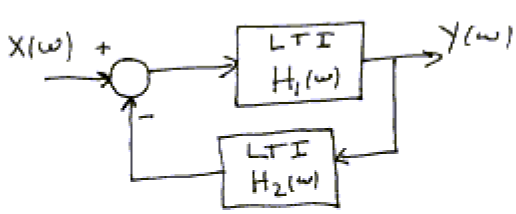




Of course, in these cases it is clear that stability of the overall system follows from stability of the individual subsystems.

The situation is more complicated for the feedback connection of stable LTI systems, but at least the Fourier transform representation permits us to achieve an explicit representation for the overall system, something that we were unable to accomplish in the time domain.

Beginning with the output, the feedback connection below gives the following algebraic relationship between the Fourier transforms of the input and output signals. (Again, the negative sign on the feedback line at the summer is traditional.)

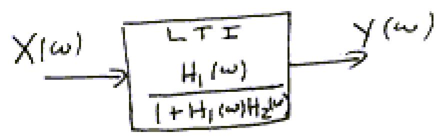


$$
\begin{aligned}
Y(\omega) &= H_1(\omega)\left[X(\omega) - H_2(\omega)Y(\omega)\right] \\
&= H_1(\omega)\,X(\omega) - H_1(\omega)H_2(\omega)Y(\omega)
\end{aligned}
$$

Solving for $Y(\omega)$ by algebraic manipulation gives

$$Y(\omega) = \frac{H_1(\omega)}{1 + H_1(\omega)H_2(\omega)} X(\omega)$$

That is, the feedback connection above is equivalent to



Of course the overall system, called the *closed-loop system* in this context, must be stable for the frequency response function shown to be meaningful. Unfortunately, the feedback connection of stable systems does not always yield a stable closed-loop system, so that further pursuit of this topic first requires the development of stability criteria for feedback systems.

Example If

$$H_1(\omega) = \frac{3}{2 + j\omega}, \quad H_2(\omega) = k$$

where k is a constant, then the frequency response of the closed-loop system is

$$H_{cl}(\omega) = \frac{3/(2 + j\omega)}{1 + 3k/(2 + j\omega)} = \frac{3}{(3k+2) + j\omega}$$

This is a valid frequency response if $k > -2/3$, in which case

$$h_{cl}(t) = 3\,e^{-(3k+2)t}u(t)$$

Indeed, by choice of k we can achieve arbitrarily fast exponential decay of the closed-loop system's unit-impulse response!