Notes for Signals and Systems

1.4 Energy and Power Classifications

The *total energy* of a continuous-time signal $x(t)$, where $x(t)$ is defined for $-\infty < t < \infty$, is

$$E_\infty = \int_{-\infty}^{\infty} x^2(t)dt = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\, dt$$

In many situations, this quantity is proportional to a physical notion of energy, for example, if $x(t)$ is the current through, or voltage across, a resistor. If a signal has finite energy, then the signal values must approach zero as t approaches positive and negative infinity.

The *time-average power* of a signal is

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

For example the constant signal $x(t) = 1$ (for all t) has time-average power of unity.

With these definitions, we can place most, but not all, continuous-time signals into one of two classes:

- An *energy signal* is a signal with finite E_∞. For example, $x(t) = e^{-|t|}$, and, trivially, $x(t) = 0$, for all t are energy signals. For an energy signal, $P_\infty = 0$.
- A *power signal* is a signal with finite, nonzero P_∞. An example is $x(t) = 1$, for all t, though more interesting examples are not obvious and require analysis. For a power signal, $E_\infty = \infty$.

Example Most would suspect that $x(t) = \sin(t)$ is not an energy signal, but in any case we first compute

$$\int_{-T}^{T} \sin^2(t)\, dt = \int_{-T}^{T} \left(\tfrac{1}{2} - \tfrac{1}{2}\cos(2t)\right) dt = T - \tfrac{1}{2}\sin(2T)$$

Letting $T \to \infty$ confirms our suspicions, since the limit doesn't exist. The second step of the power-signal calculation gives

$$P_\infty = \lim_{T\to\infty} \tfrac{1}{2T}\left(T - \tfrac{1}{2}\sin(2T)\right) = \tfrac{1}{2}$$

and we conclude that $x(t)$ is a power signal.

Example The unit-step function, defined by

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

is a power signal, since

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)\, dt = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} 1\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T} \frac{T}{2} = \frac{1}{2}$$

Example There are signals that belong to neither of these classes. For example, $x(t) = e^t$ is a signal with both E_∞ and P_∞ infinite. A more unusual example is

$$x(t) = \begin{cases} t^{-1/2}, & t \geq 1 \\ 0, & t < 1 \end{cases}$$

This signal has infinite energy but zero average power.

The *RMS* (*root-mean-square*) value of a power signal $x(t)$ is defined as $\sqrt{P_\infty}$. These energy and power definitions also can be used for complex-valued signals, in which case we replace $x^2(t)$ by $|x(t)|^2$.