

H7-08

7. Determine if the following statements about CT LTI systems are true or false. **Justify your answers.**

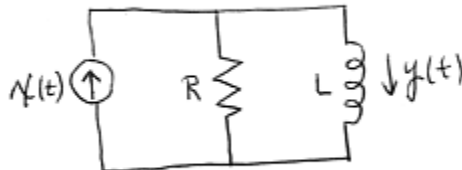
- (a) If $h(t)$ is right sided and bounded, then the system is stable.
- (b) If $h(t)$ is periodic and not identically zero, then the system is unstable.
- (c) The cascade connection of a causal LTI system and a non-causal LTI system is always non-causal.
- (d) A memoryless LTI system is always stable.

8. Consider a system described by

$$y(t) = \int_{-\infty}^t e^{-2(t-\tau)} x(\tau-1) d\tau$$

- (a) Show that this is an LTI system.
- (b) Compute the unit-impulse response of the system.
- (c) Compute the response of the system to $x(t) = u(t) - u(t-1)$ by convolution and then by using the fact that the unit-step response of an LTI system is the running integral of the unit-impulse response..

9. For the R-L circuit shown below, with input and output current signals as shown, and $R = 4$, $L = 4$, compute



- (a) the steady-state response to the input signal $x(t) = 3 \cos(t)u(t)$.
- (b) the steady-state response to $x(t) = 2 \sin(3t)u(t)$.
- (c) the response to $x(t) = 1$.

10. For the LTI system with unit-impulse response $h(t) = e^t u(t)$, compute the response to the input signal $x(t) = e^{3t} \cos(2t)$. (*Hint: We did not discuss all the eigenfunction properties that LTI systems have.*)

1. Show that the basis set that is defined on $-1 \leq t \leq 1$ by

$$\phi_0(t) = 1, \quad \phi_1(t) = t, \quad \phi_2(t) = \frac{3}{2}t^2 - \frac{1}{2}$$

with the signals zero outside of this interval, is an orthogonal basis set on the interval $-\infty < t < \infty$. If we change the third signal to $\phi_2(t) = t^2$, $-1 \leq t \leq 1$, is the new basis set orthogonal on $-\infty < t < \infty$.

2. (a) Show that for any $x(t)$,

$$x_{ev}(t) = Ev\{x(t)\}, \quad x_{od}(t) = Od\{x(t)\}$$

form an orthogonal basis set over any interval of the form $-T \leq t \leq T$.

(b) Suppose $\phi_k(t)$, $k = 0, \dots, K-1$ is an orthogonal basis set on the interval $-1 \leq t \leq 1$. Let $\varphi_k(t) = \phi_k(t/3)$, $k = 0, \dots, K-1$. On what interval, $-T \leq t \leq T$, if any, is $\varphi_k(t)$, $k = 0, \dots, K-1$ an orthogonal set?

(c) Suppose $\phi_k(t)$, $k = 0, \dots, K-1$ is an orthogonal basis set on the interval $0 \leq t \leq 1$. Let $\varphi_k(t) = \phi_k(3t-1)$, $k = 0, \dots, K-1$. On what time interval, if any, is $\varphi_k(t)$, $k = 0, \dots, K-1$ an orthogonal set?

3. Consider the set of signals defined as shown for $0 \leq t \leq 4$, and defined to be zero outside this interval:

$$\phi_0(t) = \sin\left(\frac{\pi}{2}t\right), \quad \phi_1(t) = u(t-1) - u(t-3), \quad \phi_2(t) = r(t) - 2r(t-2)$$

Is this an orthogonal set on the interval $0 \leq t \leq 4$?