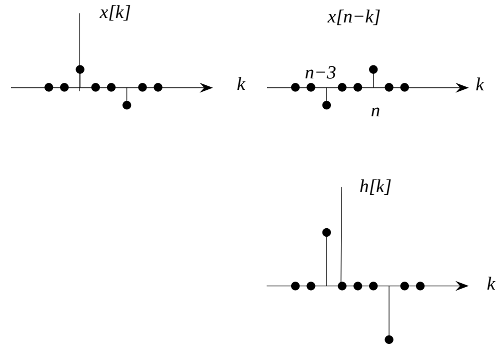
H5-08 Solutions

Problem 2a:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$



Clearly, $y[n] = 0$ for $n < -1$ and $n > 6$ (no overlap).
Picking "n", multiplying and adding gives

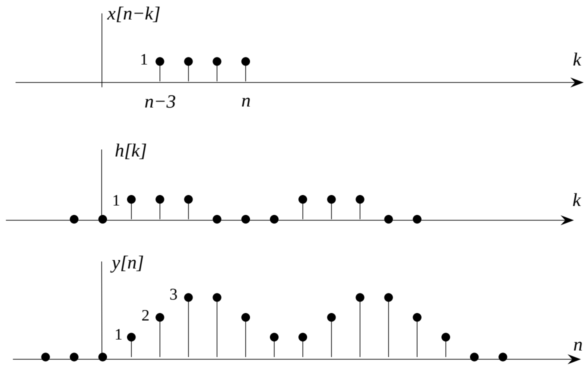
$$\begin{aligned}
&y[-1] = 3, \\
&y[0] = y[1] = 0, \\
&y[2] = y[3] = -3, \\
&y[4] = y[5] = 0, \\
&y[6] = 3
\end{aligned}$$

That is,

$$y[n] = 3\delta[n+1] - 3\delta[n-2] - 3\delta[n-3] + 3\delta[n-6]$$

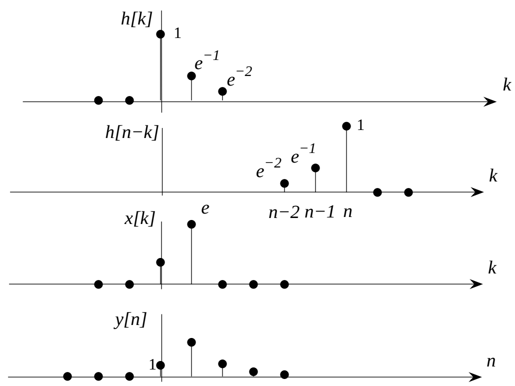
Problem 2b:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$



Problem 2e:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

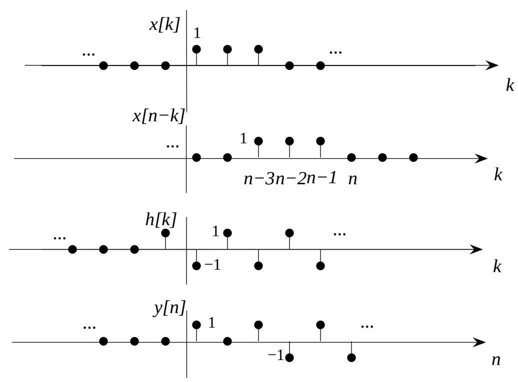


$y[n] = 0$ for $n < 0$ or (no overlap)

$$
\begin{aligned}
y[0] &= 1, \\
y[1] &= e + e^{-1}, \\
&= e(1 + e^{-2}), \\
y[2] &= 1 + e^{-2}, \\
y[3] &= e^{-1} + e^{-3}, \\
&= e^{-1}(1 + e^{-2}), \\
&\vdots \\
y[n] &= e^{(n-2)}(1 + e^{-2}), \qquad for\ n \geq 1
\end{aligned}
$$

Problem 2i:

$$
\begin{aligned}
x[n] &= u[n-1] - u[n-4] \\
h[n] &= (-1)^n u[n] \\
y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k]
\end{aligned}
$$



$y[n] = 0$ for $n \leq 0$ or (no overlap)

$$
\begin{aligned}
y[1] &= 1, \\
y[2] &= 1 - 1 = 0, \\
y[3] &= 1 - 1 + 1 = 1, \\
y[4] &= -1 + 1 - 1 = -1, \\
&\vdots \\
y[n] &= -(-1)^n, \qquad for\ n \geq 3
\end{aligned}
$$

Problem 3a:

$$
\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k] \\
&= \sum_{k=0}^{\infty} \alpha^k \beta^{n-k} u[n-k] \\
&= \begin{cases} 0, & n < 0 \\ \beta^n \sum_{k=0}^{n} (\frac{\alpha}{\beta})^k, & n \geq 0 \end{cases} \\
&= \begin{cases} 0, & n < 0 \\ \beta^n (\frac{1-(\frac{\alpha}{\beta})^{n+1}}{1-\frac{\alpha}{\beta}}), & n \geq 0 \end{cases} \\
&= \begin{cases} 0, & n < 0 \\ \frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}, & n \geq 0 \end{cases} \\
&= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} u[n]
\end{aligned}
$$

Problem 3b:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} \beta^k u[-k]\alpha^{n-k} \\ &= \alpha^n \sum_{k=-\infty}^{0} \beta^k \alpha^{-k} \end{aligned}$$

Let $j = -k$

$$\begin{aligned} &= \alpha^n \sum_{j=0}^{\infty} (\frac{\alpha}{\beta})^j \\ &= \frac{\alpha^n}{1-\frac{\alpha}{\beta}} \\ &= \frac{\beta\alpha^n}{\beta-\alpha} \end{aligned}$$

This last step requires $\mid \frac{\alpha}{\beta} \mid < 1$

Problem 7a:

$$\begin{aligned} \hat{y}[n] &= \sum_{k=-\infty}^{\infty} \hat{x}[k]\hat{h}[n-k] \\ &= \sum_{k=-\infty}^{\infty} x[k-3]h[n+3-k] \end{aligned}$$

Let $l = k - 3$

$$\begin{aligned} \hat{y}[n] &= \sum_{l=-\infty}^{\infty} x[l]h[n-l] \\ &= y[n] \end{aligned}$$

Problem 7b:

$$\begin{aligned} \hat{y}[n] &= \sum_{k=-\infty}^{\infty} \hat{x}[k]\hat{h}[n-k] \\ &= \sum_{k=-\infty}^{\infty} x[k-3]h[n-3-k] \end{aligned}$$

Let $l = k - 3$

$$\begin{aligned}\hat{y}[n] &= \sum_{l=-\infty}^{\infty} x[l]h[n-6-l] \\ &= y[n-6]\end{aligned}$$

Problem 7c:

$$\begin{aligned}\hat{y}[n] &= \sum_{k=-\infty}^{\infty} \hat{x}[k]\hat{h}[n-k] \\ &= \sum_{k=-\infty}^{\infty} x[-k]h[-n+k]\end{aligned}$$

Let $l = -k$

$$\begin{aligned}\hat{y}[n] &= \sum_{l=-\infty}^{\infty} x[l]h[-n-l] \\ &= y[-n]\end{aligned}$$

Problem 7d:

$$\begin{aligned}\hat{y}[n] &= \sum_{k=-\infty}^{\infty} \hat{h}[k]\hat{x}[n-k] \\ &= \sum_{k=-\infty}^{\infty} h[1-k]x[-1-(n-k)]\end{aligned}$$

Let $l = 1 - k$

$$\begin{aligned}\hat{y}[n] &= \sum_{l=-\infty}^{\infty} h[l]x[-1-n+1-l] \\ &= \sum_{l=-\infty}^{\infty} h[l]x[-n-l] \\ &= y[-n]\end{aligned}$$