H4-08

1. Consider the signal $x[n] = c_1 e^{j\omega_1 n} + c_2 e^{j\omega_2 n}$ where both frequencies are rational multiples of 2π ,

$$\omega_1 = (m_1 / N_1)2\pi \ , \quad \omega_2 = (m_2 / N_2)2\pi$$

Suppose that N is a positive integer such that

$$N = k_1 N_1 = k_2 N_2$$

for some integers k_1 , k_2 . Show that $x[n]$ has period N . (Typically $N < N_1 N_2$, as used in the theorem in Section 3.1.)

2. Determine if each of the following systems is causal, memoryless, time invariant, linear, or stable. **Justify your answers!**

(b) $y(t) = \cos^2(t)\, x(t)$

(d) $y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau) d\tau$

(f) $y(t) = x(-t)$

(h) $y(t) = \int_{-\infty}^{t} e^{(t-\sigma)} x^2(\sigma)\, d\sigma$

(j) $y(t) = 3x(t+1) - 4$

(l) $y(t) = 3x(t) - |\, x(t-3)\,|$

3. Determine if each of the following systems is causal, memoryless, time invariant, linear, or stable. **Justify your answers!**

(a) $y[n] = 3x[n]x[n-1]$

(c) $y[n] = 4\, x[3n-2]$

(e) $y[n] = \sum_{k=n-3}^{n} \cos(x[k])$

4. Determine if each of the following systems is invertible. If not, specify two different input signals that yield the same output. If so, give an expression for the inverse system.

(a) $y[n] = \sum_{k=-\infty}^{n} x[k]$

(b) $y[n] = (n-1)x[n]$

(c) $y[n] = x[n] - x[n-1]$

5. For each pair of systems S_1, S_2 specified below, give a mathematical description of the cascade connection $S_2(S_1)$.

(a) $y_1[n] = x_1^2[n-2], \quad y_2[n] = 3x_2[n+2]$

(b) $y_1[n] = \sum_{k=-\infty}^{n} \delta[k]x_1[n-k], \quad y_2[n] = \sum_{k=-\infty}^{n} 2\delta[n-k]x_2[k]$

6. Suppose an LTI system with input signal $x[n] = u[n] - u[n-2]$ has the response $y[n] = 2r[n] - 2r[n-2]$. Sketch this input signal and output signal, and also sketch the system response to each of the input signals below.

(b) $x_b[n] = u[n] - u[n-1] - u[n-2] + u[n-3]$

(c) $x_c[n] = u[n] - u[n-4]$

TA: 2 Pts for Problem #1. One-half point for each subpart in problems 2-6. Total of 10 points to be allocated.