

H11-08 Solutions

Pr 5:

(a)

$$\begin{aligned}\hat{N}_o &= N_o, \hat{\omega}_o \\ \hat{x}_k &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} \hat{x}[n] e^{-jk\omega_o n} \\ &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} x[n + \frac{N_o}{2}] e^{-jk\omega_o n}\end{aligned}$$

Let $m = n + \frac{N_o}{2}$

$$\begin{aligned}\hat{x}_k &= \frac{1}{N_o} \sum_{m=N_o/2}^{N_o/2+N_o-1} x[m] e^{-jk\omega_o(m-N_o/2)} \\ &= e^{-jk\omega_o \frac{N_o}{2}} \frac{N_o}{2} \sum_{m=\langle N_o \rangle} x[m] e^{-jk\omega_o m}\end{aligned}$$

But $\omega_o \frac{N_o}{2} = \pi$, so

$$\hat{x}_k = (-1)^k x_k$$

(b) Assume $\hat{N}_o = N_o, \hat{\omega}_o = \omega_o$. (This might not be the case, e.g. $x[n] = (-1)^n$). Then

$$\begin{aligned}\hat{x}_k &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} (-1)^n x[n] e^{-jk\omega_o n} \\ &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} x[n] e^{j\pi n} e^{-jk\omega_o n} \\ &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} x[n] e^{-j\frac{\omega_o N_o}{2} n} e^{jk\omega_o n} \\ &= \frac{1}{N_o} \sum_{n=0}^{N_o-1} x[n] e^{-j(k-\frac{N_o}{2})\omega_o n} \\ &= x_{k-\frac{N_o}{2}}\end{aligned}$$

Pr 1:

(a)

$$\begin{aligned}X(\omega) &= \int_{-\infty}^{\infty} e^{-(t-2)} u(t-3) e^{-j\omega t} dt = e^2 \int_3^{\infty} e^{-(1+j\omega)t} dt \\ &= \frac{-e^2}{1+j\omega} e^{-(1+j\omega)t} \Big|_3^{\infty} = \frac{e^2}{1+j\omega} e^{-(1+j\omega)3} \\ &= \frac{e^{-1} e^{-j\omega 3}}{1+j\omega}\end{aligned}$$

(b)

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} e^{-|t+1|} e^{-j\omega t} dt = \int_{-1}^{\infty} e^{-(t+1)} e^{-j\omega t} dt + \int_{-\infty}^{-1} e^{t+1} e^{-j\omega t} dt \\ &= e^{-1} \int_{-1}^{\infty} e^{-(1+j\omega)t} dt + e \int_{-\infty}^{-1} e^{(1-j\omega)t} dt \\ &= \frac{-e^{-1}}{1+j\omega} e^{1+j\omega} + \frac{e}{1-j\omega} e^{-1+j\omega} = \frac{2e^{j\omega}}{1+\omega^2} \end{aligned}$$

(c)

$$\begin{aligned} X(\omega) &= \int_0^1 2e^{-j\omega t} dt + \int_1^{\infty} 2e^{-(t-1)} e^{-j\omega t} dt \\ &= \frac{-2}{j\omega} e^{-j\omega t} \Big|_0^1 + 2e \int_1^{\infty} e^{-(1+j\omega)t} dt \\ &= 2 \frac{1 - e^{-j\omega}}{j\omega} + \frac{2e^{-j\omega}}{1+j\omega} \\ &= 2 \frac{1 + j\omega - e^{-j\omega} + j\omega e^{-j\omega}}{j\omega(1+j\omega)} \end{aligned}$$

(d)

$$\begin{aligned} x(t) &= \sum_{m=0}^{\infty} a^m \delta(t-m) \Rightarrow \\ X(\omega) &= \int_{-\infty}^{\infty} \left(\sum_{m=0}^{\infty} a^m \delta(t-m) \right) e^{-j\omega t} dt \\ &= \sum_{m=0}^{\infty} a^m \int_{-\infty}^{\infty} \delta(t-m) e^{-j\omega t} dt \\ &= \sum_{m=0}^{\infty} a^m e^{-j\omega m} = \sum_{m=0}^{\infty} (ae^{-j\omega})^m \\ &= \frac{1}{1 - ae^{-j\omega}} \quad , \text{ since } |a| < 1 \end{aligned}$$

Pr 2:

(a)

$$\begin{aligned} x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \int_{-\infty}^{\infty} e^{-|\omega|} e^{j\omega t} d\omega \\ &= \int_{-\infty}^0 e^{(1+jt)\omega} d\omega + \int_0^{\infty} e^{(-1+jt)\omega} d\omega \\ &= \frac{1}{1+jt} e^{(1+jt)\omega} \Big|_{-\infty}^0 + \frac{1}{-1+jt} e^{(-1+jt)\omega} \Big|_0^{\infty} \\ &= \frac{1}{1+jt} + \frac{-1}{-1+jt} = \frac{2}{t^2+1} \end{aligned}$$

(b)

$$\begin{aligned}x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-2}^2 2\pi e^{-j\omega} e^{j\omega t} d\omega \\&= \int_{-2}^2 e^{-j(t-1)\omega} d\omega = \frac{1}{j(t-1)} e^{-j(t-1)\omega} \Big|_{-2}^2 \\&= \frac{1}{j(t-1)} [e^{j(t-1)2} - e^{-j(t-1)2}] \\&= \frac{2}{t-1} \frac{e^{j2(t-1)} - e^{-j2(t-1)}}{2j} \\&= \frac{4 \sin(2(t-1))}{2(t-1)}\end{aligned}$$

Clearly this can also be written as a sinc function.

(d) From plot,

$$\begin{aligned}X(\omega) &= \begin{cases} -j & , -2 < \omega < 0 \\ +j & , 0 < \omega < 2 \\ 0 & , else \end{cases} \\x(t) &= -\frac{1}{2\pi} \int_{-2}^0 j e^{j\omega t} d\omega + \frac{1}{2\pi} \int_0^2 j e^{j\omega t} d\omega \\&= -\frac{1}{2\pi} \cdot \frac{j}{jt} [1 - e^{-j2t}] + \frac{1}{2\pi} \cdot \frac{j}{jt} [e^{j2t} - 1] \\&= \frac{-1 + \cos(2t)}{\pi t}\end{aligned}$$

(c)

$$\begin{aligned}X(\omega) &= e^{-\omega} u(\omega) \quad \Rightarrow \\x(t) &= \frac{1}{2\pi} \int_0^{\infty} e^{-\omega} e^{j\omega t} d\omega \\&= \frac{1}{2\pi} \int_0^{\infty} e^{(-1+jt)\omega} d\omega \\&= \frac{1}{2\pi(1-jt)}\end{aligned}$$

Pr 4:

(a)

$$\begin{aligned}x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \\ \Rightarrow x(0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega = 1 \\ \Rightarrow & \int_{-\infty}^{\infty} X(\omega) d\omega = 2\pi\end{aligned}$$

(b)

$$\begin{aligned}x(1) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega 1} d\omega = 0 \\ \Rightarrow \int_{-\infty}^{\infty} X(\omega) d\omega &= 0\end{aligned}$$

(c)

$$\begin{aligned}X(\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ \Rightarrow X(0) &= \int_{-\infty}^{\infty} x(t) dt = 0\end{aligned}$$

Pr 6: HW10-6

(a)

$$|X(\omega)| = \left| \frac{3 - j\omega}{3 + j\omega} \right| = \frac{|3 - j\omega|}{|3 + j\omega|} = 1$$

(b)

$$\begin{aligned}\angle X(\omega) &= \angle \frac{3 - j\omega}{3 + j\omega} = \angle(3 - j\omega) - \angle(3 + j\omega) \\ &= \tan^{-1}\left(-\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) \\ &= -2 \tan^{-1}\left(\frac{\omega}{3}\right)\end{aligned}$$

(c)

$$\begin{aligned}X(\omega) &= \frac{3}{3 + j\omega} - \frac{j\omega}{3 + j\omega} \\ &= 3 \cdot \frac{1}{3 + j\omega} - j\omega \cdot \frac{1}{3 + j\omega}\end{aligned}$$

Since $f^{-1}\left[\frac{1}{3 + j\omega}\right] = e^{-3t}u(t)$,

$$\begin{aligned}x(t) &= 3e^{-3t}u(t) - \underbrace{\frac{d}{dt}[e^{-3t}u(t)]}_{-3e^{-3t}u(t) + e^{-3t}\delta(t)} \\ &= 6e^{-3t}u(t) - \delta(t)\end{aligned}$$